Answers for #2

Economics 630: Mathematical Economics I
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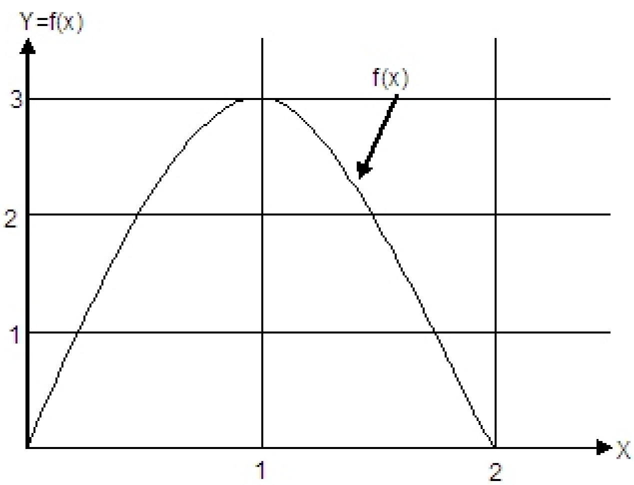
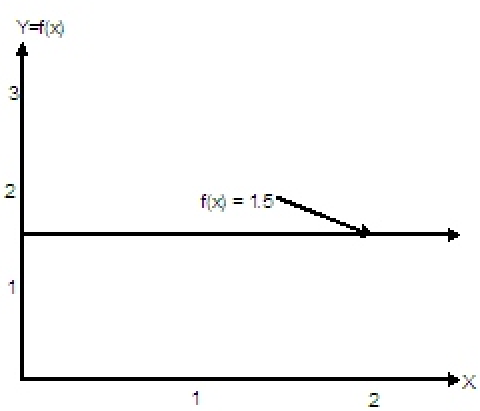
1. Sketch a graph of a function with the following properties:

a) $f'(x) > 0$ for $x < 1$; $f'(x) < 0$ for $x > 1$

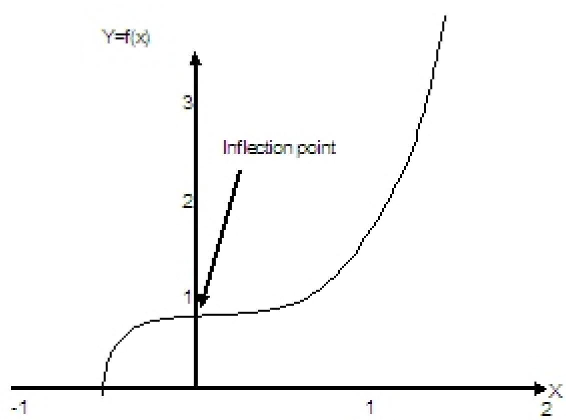


Graph of fcn where $f'(x) > 0$ for $x < 1$; $f(x) < 0$ for $x > 1$

b) and I present two candidate answers for part b): f(x) is NEVER decreasing, but $f'(x) = 0$ at $x = 0$.



First graph of fcn where f'(x) is never decreasing but $f'(x) = 0$ at $x = 0$ (and everywhere else, for that matter.)



First graph of fcn where f(x) is never decreasing but $f'(x) = 0$ at $x = 0$

2. Problem 3.9, parts e & f
For each of the following functions *f* with specified domains D_1 and D_2, find the global maximum and the global minimum of *f* on each D_i if they exist. Justify your answers.
e) $3x^5 - 5x^3$

$D_1 = [-2, +2]$

f.o.c.: $15x^4 - 15x^2 = 15x^2 (x^2 - 1) = 0$ when $x = 1$, $x = -1$, or $x = 0$. Therefore our critical points are (0,0), (−1, 2), (1, −2). These are the ONLY critical points, so 2 is the global max and −2 is the global min for f(x) defined over all of R^1.

All of these points are in the interval [−2, +2], so (−1, 2) is our global max and (1, −2) is our global min.

Aside: (0,0) must be an inflection point.

$D_2 = [-(2)^{½}, +(2)^{½})$

Answer is the same as for D_1, since $(2)^{½}$ is about 1.3, which is greater than 1.

3. Problem 3.18
Prove that the point elasticity is −1 exactly at the midpoint of the linear demand in figure 3.20.

Plan – Use the definition of elasticity, the formula for the demand function shown in fig 3.20, and the midpoint.

Step 1: Find the formula for the demand function. Note that figure 3.20 is given "mathematically properly," with Price on the horizontal axis and Quantity on the vertical axis. The formula for a line is y = mx + b, where m is the slope and b is the vertical (in this case the "x") intercept. b is, as you can see, b. the slope is $\Delta y/\Delta x$, which is $-b/(a/b)$, or $-b^2/a$. (Remember to make the slope negative)

Demand: $x = -(b^2/a)p + b$.

Step 2: Find the midpoint: Well, by inspection it will be b/2, but just to check our formula: When $p = a/2b$, $x = -(b^2/a)(a/2b) + b = -b/2 + b = b/2$.

Definition of point elasticity: (dx/dp)(p/x)
$dx/dp = -(b^2/a)$. So Elasticity evaluated at the midpoint is $-(b^2/a)(a/2b)(2/b) = -1$. q.e.d.

4. Problems 4.1 and 4.3, parts d & e.
4.1: For each of the following pairs of functions g and h, write out the composite functions g(h(x)) and h(g(x)) in as simple a form as possible. In each case, describe the domain of the composite.
4.3 Use the chain rule to compute the derivative of the composite functions AND compute the derivative using your simplified expression. Check that they are the same.

Comments – this problem comes with a built in check, so I expect perfect grades.

d) $g(x) = 4x + 2$ $h(z) = 1/4(z - 2)$

The composite functions will be the same, since this is addition of linear functions:

$g(h(x)) = 4[1/4(x-2)] + 2 = x - 2 + 2 = x$

$h(g(x)) = 1/4[(4x + 2) - 2] = (1/4*4x = x$

Directly: $d(g(h(x))/dx = 1$ and $d(h(g(x))/dx = 1$

Chain rule: $d(g(h(x)))/dx = dg/dh * dh/dx = 4 * 1/4 = 1$
$d(h(g(x)))/dx = dh/dg * dg/dx = 1/4 * 4 = 1$

e) $g(x) = 1/x$ $h(z) = z^2 + 1$

$dg/dx = -x^{-2}$ $dh/dz = 2z$

$g(h(x)) = 1/(x^2 + 1)$ $d(g(h(x)))/dx = -2x\ (x^2 + 1)^{-2}$

and by chain rule: $dg/dh * dh/dx = -(x^2 + 1)^{-2} * 2x = -2x\ (x^2 + 1)$

$h(g(x)) = x^{-2} + 1$ $d(h(g(x)))/dx = -2x^{-3}$

and by chain rule: $dh/dg * dg/dx = 2(x^{-1}) * -x^{-2} = -2x^{-3}$

5. Problems 4.8 and 4.9, parts c & d.
c) $f(x) = x^{2/3}$
Let $f(x) = y$, so we can rewrite: $y = x^{2/3}$

then $y^{3/2} = x$ is the inverse.
Test: let x = 27. Then y = 9, and $9^{3/2} = 27$.

The inverse function theorem tested:

$dy/dx = 1/\ (dx/dy)$ $3/2 * y^{1/2} = 1/\ (2/3)x^{-1/3}$

sub x for y: $3/2 * (x^{2/3})^{½} = 3/2 * x^{1/3}$ yup. (Slang for q.e.d.)

Is there anything special about the domain over which the original function can be inverted? Well, domain of the function is $[0, \infty)$, and the function is one-to-one over the entire domain, so is invertible over its whole domain.

d) OUCH – this one is EXTRA CREDIT. I think there MUST be typo in the textbook. The problem is actually not impossible, but the clue is USELESS if this is the function.

$y = x^2 + x + 2$	This function has imaginary roots (graph never crosses x-axis!) So solving for the roots, using the quadratic formula, doesn't help set up the inverse.

But we can invert the equation by "completing the square."

Before doing so, however, consider whether the function is one-to-one over its whole domain. Answer is NO. Domain is $-\infty$ to ∞, so that is fine. But it is a parabola with a minimum at (-0.5, 1.75). If the function is defined to be only over the domain $(-\infty, -0.5]$ OR the function is defined only over the domain $[-0.5, \infty)$, then each of these two function IS invertible.

Plan: Complete the square – want to express y = f(x) as something in the form $y = (x + a)^2 + b$

$y = x^2 + x + 2 = x^2 + x + 0.25 + 1.75$

$y = (x + 0.5)^2 + 1.75$

$y - 1.75 = (x + 0.5)^2$

$(y - 1.75)^{½} = x + 0.5$

$x = (y - 1.75)^{½} - 0.5$

That is the formula for the inverse. But we must define the y-values over which this can be calculated and still be a function.

This is a function for $y \geq -0.5$ or for $y \leq -0.5$. It is not a function over any interval that contains -0.5 as an internal point.

You could ALSO do this problem graphically.