Answers #4

Economics 630: Mathematical Economics I

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1. Solution for the non-singular triangular system: $w = b_3$; $v = b_2 - b_3$; $u = b_1 - b_2$

Show your solution gives a combination of the columns that equal the column on the right.

$$\begin{array}{ccccccc} (b_1 - b_2) & + & (b_2 - b_3) & + & b_3 & = & b_1 \\ & & (b_2 - b_3) & + & b_3 & = & b_2 \\ & & & & b_3 & = & b_3 \end{array}$$

2. The system will have the solution x = y (the 45 degree line) for a value of $a = -2$.
There are no other values of a which will give us an infinite # of solutions.Proof: We need to have no pivots in the second eqn – which we find by adding the first eqn to the second and solving for zero coefficients: $(2 + a)x + (2 + a)y = 0$. Clearly the only value for which the coefficients are both zero is $a = -2$.

3. Forward (Gaussian) elimination yields:

$$\begin{array}{ccccccc} u & + & v & + & w & = & 2 \\ & & 2v & + & 2w & = & -2 \\ & & & & 2w & = & 2 \end{array}$$

Which solves to: $w = 1$; $v = -2$; $u = 3$

4. $$Ax = \begin{vmatrix} 3 & -6 & 0 \\ 0 & 2 & -2 \\ 1 & -1 & -1 \end{vmatrix} \begin{vmatrix} 2 \\ 1 \\ 1 \end{vmatrix} = \begin{matrix} 6-6+0 \\ 0+2-2 \\ 2-1-1 \end{matrix} = \begin{matrix} 0 \\ 0 \\ 0 \end{matrix}$$

Can you find more than one solution? Yes, we can find more than one solution, because the system is singular. If we try Gaussian elimination we get the matrix:

$$\begin{matrix} 3 & -6 & 0 \\ 0 & 3 & -3 \\ 0 & 0 & 0 \end{matrix}$$

Which means that we can choose any value we wish for one of the three variables and find solutions for the other two. There are an infinite number of solutions.

More specifically, the second row tells us y = z, all z; the first row tells us x = 2y, all y. Therefore we have 2 equations, 3 unknowns, and can solve for any value of z.