Answer #5

Economics 630: Mathematical Economics I
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Fall 2003, Department of Economics, George Mason University

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Worth 3 points each (24 points total)

1. Problem 6.1

This is easy to do algebraically, but I will set up the Gauss-Jordan elimination. Notice how I am free to arrange the linear equations so that when I write the matrix form the columns and rows to simplify the math:

Let F = Federal taxes paid
Let S = State taxes paid
Express all quantities in 1,000s

F = 0.4(100 - S)	F	+	0.4S	=	40
S = 0.05(100) = 5	0	+	S	=	5

Can be rewritten as the matrix eqn A**x** = **b**

$$\begin{vmatrix} 1 & 0.4 \\ 0 & 1 \end{vmatrix} \begin{vmatrix} F \\ S \end{vmatrix} = \begin{vmatrix} 40 \\ 5 \end{vmatrix}$$

Which solves to S = $5,000 and F = 38,000. Total Taxes = 43,000. After tax income = 57,000.

Therefore, compared to after tax and charity income given in Example 1 of 53,605. The charitable contribution of $5,956. (See p. 109) has an opportunity cost of only 57,000 - 53,605 = $3,395.

2. Problem 6.3

The economy on the island of Bacchus produces only grapes and wine. The production function is represented by the following three linear equations:

1 lb of grapes requires ½ a lb of grapes, no wine, and one worker
1 liter of wine requires ½ a lb of grapes, 1/4 liters of wine, and one worker.

Island Labor Supply is 10 people. Island consumption is 1 lb grapes, 3 liters wine.
(Am I the only person who wondered why grapes are in pounds, not kilos, or alternatively wine is in liters, not quarts or pints or bottles or barrels?

The Linear Equations:
Let Xw = production of wine; Cw = consumption of wine, both measured in pounds
Let Xg = production of grapes; Cg = consumption of grapes, both measured in grapes.
Let L = number of workers; -L = consumption of labor

$$Xg = 0.5\ Xg + 0.5\ Xw + Cg = 0.5\ Xg + 0.5\ Xw + 1$$

$$Xw = 0.0\ Xg + 0.25\ Xw + Cw = 0.0\ Xg + 0.25\ Xw + 3$$

$$L = -Xg - Xw = -10$$

Which we can re-arrange to give us the following sytem of n+1 equations in n unknowns:

$$\begin{aligned} 0.5\ Xg - 0.5\ Xw &= 1 \\ 0.0\ Xg + 0.75\ Xw &= 3 \\ -Xg \quad -Xw &= -10 \end{aligned}$$

We have 3 equations in only two unknowns, so the system may or may not have a solution for a given set of consumption values.

0.75 Xw = 3; Xw = 4
0.5 Xg - 2 = 1; 0.5 Xg = 3; Xg = 6
-4 -6 = -10.

Yes, this set of demands works out for a labor supply of 10. So, of course, do an infinite number of other sets of demand.

3. Problem 6.5
The Answer in the back of the book is WRONG. The MODEL we are given on page 114 gives employment in one week as a function of the unemployment in the previous WEEK. The question asks about the YEAR to YEAR variation. However the ANSWER in the back of the book is the increase in one WEEK.

Here is the one week version:
Let x = % men currently employed; q = % of the employed who STAY employed
Let y = % men currently unemployed; p = % of unemployed who BECOME employed

Therefore we can describe the week to week dynamics of average unemployment by the linear equations:

$$x_{t+1} = qx_t + py_t$$

$$y_{t+1} = (1-q)x_t + (1-p)y_t$$

After one week, for white males: $y_{t+1} = 0.002*0.9 + 0.864*0.1 = .0018 + 0.0864 = 0.0882$
And for black men, $y_{t+1} = 0.004*0.8 + 0.898*0.2 = 0.1828$

Doing it right is much harder. Consider the equation for TWO WEEKS later:

$$y_{t+2} = (1-q)x_{t+1} + (1-p)y_{t+1} = (1-q)[(1-q)x_t + (1-p)y_t] + (1-p)(qx_t + py_t)$$

As you can see, this gets complicated real fast.

If we ASSUME that the model converges, 52 iterations is going to get us very close to the long run equilibrium of 1.4 for whites and 3.77 for blacks.

Full points will be awarded for either the book answer OR for trying to answer the one-year problem.

4. Problem 7.3 b and c.
Solve by Gauss-Jordan elimination:

b)

4	2	−3	|	1
6	3	−5	|	0
1	1	2	|	9

the row echelon form with normalized pivots:

1	0.5	−0.75	|	0.25	
0	1	5.5	|	17.5	
0	0	1	|	3	2nd and 3rd rows switched.

And when we carry out Jordan elimination:

1	0	0	|	-2
0	1	0	|	1
0	0	1	|	3

So x = -2, y = 1, z = 3.

And c)

2	2	−1	|	2
1	1	1	|	−2
2	−4	3	|	0

swap the 2nd and 3rd rows, and the row echelon with normalized pivots is:

1	1	-0.5	|	1
0	1	-2/3	|	1/3
0	0	1	|	-2

Reverse elimination yields:

1	0	0	|	1
0	1	0	|	-1
0	0	1	|	-2

5. Problem 7.8: Solve for the general solution for:

$$a_{11}x_1 + a_{12}x_2 = b_1$$
$$a_{21}x_1 + a_{22}x_2 = b_2$$

$x_2 = (b_2 - a_{21}x_1) / a_{22}$

substituting into the first equation and solving for x_1 yields:

$$x_1 = (a_{22}b_1 - a_{12}b_2) / (a_{11}a_{22} - a_{12}a_{21})$$

And if we solve the first eqn for x_1 and substitute into equation 2 and solve for x_2 we get:

$$x_2 = (a_{11}b_2 - a_{21}b_1) / (a_{11}a_{22} - a_{12}a_{21})$$

Both solutions have the same denominator, so $(a_{11}a_{22} - a_{12}a_{21}) \neq 0$ is a necessary condition in order for a solution to exist. (We will see later that this is the determinant of a 2x2 matrix).

6. Problem 7.21 iii and iv

iii. a) Rank = number of columns = 2, one solution: $x = y = 0$
b) Rank = number of columns = 2, so either one or no solutions.

iv. a) Rank = number rows = number columns, so there is a unique solution: $x = y = z = 0$
b) Rank = number rows = number columns, so there is a unique solution.

7. Problem 7.25

i) Two equations, 4 unknowns, there will be 2 exogenous variables. The way the problem is currently set up, Gauss-Jordan elimination creates the following matrix of coefficients:

1	2	0	−1	|	0.75
0	0	1	0	|	0.25

Pivots are found in columns 1 and 3. Rank of the system is 2. The z variable is always fixed, at 0.25, so it is always endogenous. Any two of the other 3 variables can be freely picked (exogenous) and this will determine the value of the final variable.

ii) The math on this one is a pain, but eventually one gets to:

1	0	-1	0	|	1
0	1	1	0	|	0
0	0	0	1	|	0

This matrix has rank of 3, (3 rows, 3 pivots); 4 variables, so one can be picked freely. w is fixed at zero. Value of x, y or z can be picked, then the other 2 are fixed.

8. Problem 8.20 a, b and c.

a)

2	1	|	1	0
1	1	|	0	1

Gauss-Jordan elimination of the augmented matrix above yields:

1	0	|	1	−1
0	1	|	−1	2

And **x** = A^{-1} * **b**, so our solution is: x = 5 - 3 = 2
y = -5 + 6 = 1

b) There is a typographic error in the answer key in the back of the book. a_{31} = 5/3, not 5/2

A^{-1} =

−6	3/2	−1
13	−3	2
5/3	−1/3	1/3

A^{-1} * **b**;
b = (4, 20, 3)

x = -6*4 + (3/2)*20 - 1*3 = −24 + 30 − 3 = 3
y = 13*4 −3*20 + 2*3 = 52 - 60 + 6 = −2
z = (5/3)*4 −(1/3)*20 + (1/3)*3 = 20/3 - 20/3 + 1 = 1

c) A^{-1} =

−5/2	0	−1
3/2	0	½
1/3	1/3	1/3

A^{-1} * **b**;
b = (2, 1, 6)

x = 2*(-5/2) + 1*0 −1*6 = −5 + 0 + 6 = 1
y = (3/2)*3 + 0*1 + (½)*−6 = 3 + 0 −3 = 0
z = (1/3)*2 + (1/3)*1 −(1/3)*(6) = 2/3 + 1/3 - 2 = −1