Problem set #6

Economics 630: Mathematical Economics I

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Last Problem Set before the Midterm

1. Problem 9.4
Show that one obtains the same formula for the determinant of a 2x2 matrix, no mater which row or column one uses for the expansion.

Let the 2x2 matrix be represented by:

A	B
C	D

Using first row: $(-1)^{1+1}$ AD + $(-1)^{1+2}$ BC = AD - BC
Using second row: $(-1)^{2+1}$ CB+$(-1)^{2+2}$ AD = -CB + AD
Using first column: $(-1)^{1+1}$ AD + $(-1)^{2+1}$ CB + AD - CB
Using second column: $(-1)^{1+2}$ BC + $(-1)^{2+2}$ DA = -BC + DA

2. Problem 9.5
Us a formula for the determinant to verify Theorem 9.1 for upper triangular 3x3 matrices:

Theo. 9.1
The determinant of a lower-triangular, upper triangular, or diagonal matrix is simply the product of its diagonal entries:

A	b	c
0	D	e
0	0	F

A*(DF-e0) −b*(0F - e0) + c(0*0 - D0) = ADF

3. Problems 9.11 and 9.12, a) and c). NOT B.
9.11 - Use Theo 9.4 to invert the following matrices:

a)

4	3
1	1

c)

a	b
c	d

9.12 (HINT – you can do matrix algebra in excel to check yourself. Look at the mdeterm function.)

1	1	1	x_1		0
12	2	−3	x_2	=	5
3	4	1	x_3		−4

Cramer's rule: x_i = det B_i / det A

We have already determined that $x_3 = 1$.

det A = 1

det B_1 = det

0	1	1
5	2	−3
−4	4	1

0*(2+12) −1*(5−12) + 1*(20+8) = 35. x_1 = 35/35 = 1, same as x_3.

det B_2 = det

1	0	1		
12	5	−3	=	−70
3	−4	1		

Therefore x_3 = −70/35 = 2.

Check: The three numbers work in the three equations.

4. Problem 26.19 ("Show" means "lay out the intuition of the proof to your own satisfaction," don't worry about formally proving, just convince me and yourself.)

Show that the determinant of A is, up to sign, the product of its pivots.

We know, by fact 26.8, that if R is the row echelon form of the matrix, then det R = ±det A.

And we know, by fact 26.11, that the determinant of an upper- or lower triangular matrix is the product of its diagonal entries.

And we know that the row echelon form of a matrix is upper triangular, and the diagonal entries are either pivots or zero. Therefore the determinant of the row echelon matrix is the product of the diagonal, and it is the product of its pivots OR it is zero, if one of the pivots is missing.

5. Problem 26.22

Use row reduction to show that

$$\text{Det} \begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^2 & b^2 & c^2 \end{vmatrix} = (b - a)(c - a)(c - b)$$

The top row is multiplied by a and subtracted from the second row

$$\begin{vmatrix} 1 & 1 & 1 \\ 0 & b-a & c-a \\ a^2 & b^2 & c^2 \end{vmatrix}$$

The top row is multiplied by a^2 and subtracted from the last row:

$$\begin{vmatrix} 1 & 1 & 1 \\ 0 & b-a & c-a \\ 0 & b^2-a^2 & c^2-a^2 \end{vmatrix}$$

which can be re-expressed:

$$\begin{vmatrix} 1 & 1 & 1 \\ 0 & b-a & c-a \\ 0 & (b-a)(b+a) & (c-a)(c+a) \end{vmatrix}$$

And now we can multiply the second row by (b+a)

$$\begin{vmatrix} 1 & 1 & 1 \\ 0 & b-a & c-a \\ 0 & 0 & (c-a)(c+a) - (c-a)(b+a) \end{vmatrix}$$

Which can be re-expressed as:

$$\begin{vmatrix} 1 & 1 & 1 \\ 0 & b-a & c-a \\ 0 & 0 & (c - a)(c - b) \end{vmatrix}$$

And the product of the diagonals is (b-a)(c-a)(c-b). Q.e.d.