Problem set #10

Economics 630: Mathematical Economics I
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This problem set is worth 24 points (4 points each)

1. Problem 14.9

A) Q(K = 1000, K = 125) = 3 * 100 * 5 = 1500

B) $\partial Q/\partial K = 3*(2/3)*(L/K)^{1/3} = 2 * (5/10) = 1$

$\partial Q/\partial L = 3 * (1/3) * (K/L)^{2/3} = 1 * (100/25) = 4$

Therefore

$Q(K = 998, L = 128) \approx Q(K = 1000, L = 125) + \partial Q/\partial K * (-2) + \partial Q/\partial L * (3)$
$= 1500 - 2 + 12 = 1510.$

2. Problem 14.11

a) $df/dt = \partial f/\partial x*dx/dt + \partial f/\partial y*dy/dt = 3y(t)$

$\partial f/\partial x = 3y^2 + 2$ $\qquad dx/dt = -6t$

$\partial f/dy = 6xy$ $\qquad dy/dt = 12t^2 + 1$

$df/dt = 3(4t^3 + t) * (-6t) + 6(-3t^2)(4t^3 + t) * (12t^2 + 1)$

b) The direct substitution yields:

$f(t) = 3(-3t^2)(4t^3 + t)^2 + 2(-3t^2)$

multiply the terms out and take the first derivative wrt t and compare to the answer in a)

3. Problem 15.6

a) $y = -3$

b) $\partial F/\partial y(6, 3, -3) = 27 \neq 0$; so the equation implicitly defines y as a function of x around the point (6, 3, – 3).

c) $\partial y/\partial x_1\,(6, 3) = -(\partial F/\partial x_1) / (\partial F/\partial y) = -(2x_1 / 3y^2) = -4/9$

$\partial y/\partial x_2\,(6, 3) = -(\partial F/\partial x_2) / (\partial F/\partial y) = (2x_2 / 3y^2) = 2/9$

d) $y(6.2, 2.9) \approx y(6, 3) - (4/9)(0.2) + (2/9)(-0.1) = -28/9$

4. Problem 15.31 and 15.32

What follows is not a complete answer, but the intuition for the answer:

15.31) Work through section 15.4 up to equation (49) on p. 362. At this point the book works through the comparative statics for what happens if we alter e_2, leaving e_1 the same. (Change wealth of individual 2, leave individual one alone.)

We want to see what happens to our five variables, x_1, x_2, y_1, y_2, and p (and therefore to U_1 and U_2, which are functions of consumption) if we change the wealth of individual #1, while leaving individual #2's endowment unchanged.

We pick up the problem at equation (49). The only difference in the next step is that we solve the last two equations for dx_1 and dy_1, and substitute so we have a system of equations in terms of individual 2's consumption.

There are a lot of symmetries here – so we expect to get something which looks a lot like equation (50), which can then be solved using Cramer's rule bo get expressions for dx_2, dy_2, and dp. (NOTE – dq will always be 0 since q is the "numeraire," fixed at 1, also note that with fixed total quantities of x and y, this system also determines $dx_1 = -dx_2$ and $dy_1 = -dy_2$.)

And we determine the actual partials by using the implicit function theorem – the results will look a lot like equations (53) in the book, by symmentry.

15.32 Comparative statics of increasing α: We are looking for $\partial x_1/\partial\alpha$, $\partial x_2/\partial\alpha$, $\partial y_1/\partial\alpha$, $\partial y_2/\partial\alpha$, $\partial p/\partial\alpha$ that result from changing α. This is a lot easier than 15.31 because α does not show up inside the utility functions, u_i and u_2.

Intuition – when α increases, it means that the first good, x, is more valuable and the second good, y, is less valuable. This means that the first individual, 1, will also be richer, and 2 will be poorer, and p will go up. We predict $\partial p/\partial\alpha > 0$. It is harder to predict what happens to the distribution of x and y, because that depends on whether they are both normal, or one is inferior (they can't both be inferior). Individual #1 will consume more of normal goods, and less of inferior goods, and vice versa for individual 2.

5. Problem 17.1

A) 6 critical points: (0,0), (0,1), (1,0), (– 1, 0), (+1/(5)^{½,} 2/5) (– 1/5$^{1/2}$, 2/5)

The Hessian H = $\begin{matrix} 6xy & 2y + 3x^2 - 1 \\ 2y + 3x^2 - 1 & 2x \end{matrix}$

Which we evaluate for each of the six points:
(0,0) – indefinite, saddle
(0,1) – indefinite, saddle
(1,0) – indefinite, saddle
(– 1, 0) – indefinite, saddle
(+1/(5)^{½,} 2/5) – positive definite, a local minimum
(– 1/5$^{1/2}$, 2/5) – negative definite, a local maximum

B) Critical Point: (13/7, 16/7)

Hessian = $\begin{matrix} 2 & -6 \\ -6 & 4 \end{matrix}$

this is indefinite, so the point is a saddle.

C) Three critical points: (– 1, – 1), (0,0), (1,1)

Hessian = $\begin{matrix} 12x^2 + 2 & -6 \\ -6 & 6 \end{matrix}$

(– 1, – 1) – H is positive definite, so a local min
(1, 1) – H is positive definite, so a local min
(0, 0) – H is indefinite, so a saddle

D) 3 critical points: (0, 0), (– ½, – ½), (½, – ½)

(0, 0) – H is indeterminate, but f(0, y) = $-y^3$, so (0, 0) is neither a max nor a min

(– ½, – ½) – H is pos def, so point is a local min

(½, – ½) – H is pos def, so point is a local min.

6. Problem 17.4

The function that the firm is maximizing is: Profit = Total Revenue - Total Costs

Profit = $\pi = Q*P_Q - x * P_x - y * P_y$

$= (xy)^{1/4} - 4x - 4y$

f.o.c. $\partial\pi/\partial x = 0.25\ (x)^{-3/4}\ (y)^{1/4} - 4 = 0$

$\partial\pi/\partial y = 0.25\ (x)^{1/4}\ (y)^{-3/4} - 4 = 0$

Which solves to give $x = y = 1 / 256$

s.o.c. The second order conditions are that the Hessian (matrix of second order partials) be negative definite – or that the first minor be negative and the second minor be positive.

I am not going to work out the Hessian (matrix of 2^{nd} partials) and minors here, but it turns out that the Hessian is negative definite for all $(x,y) \in R^2$