

Name:

Econ 630, Fall 2003. Final

e-mail address: _____

Phone number: _____

General Instructions:

READ EACH QUESTION CAREFULLY. PARTIAL CREDIT WILL NOT BE GIVEN TO STUDENTS WHO FAIL TO GIVE THE CORRECT ANSWER BECAUSE THEY MISREAD THE QUESTION.

PUT YOUR NAME ON ALL OF THE SHEETS OF THE EXAM BOOKLET, INCLUDING THE SCRATCH PAPER AND THE GRAPH PAPER. DO ALL WORK IN THE EXAM BOOKLET.

CALCULATORS, SLIDE RULES, PENS, PENCILS, ERASERS, AND RULERS ARE THE ONLY TOOLS YOU MAY USE ON THIS EXAM.

PLEASE READ AND SIGN THE FOLLOWING:

I understand that academic integrity is highly valued at GMU. Further, I understand that academic dishonesty, such as cheating and plagiarism, are violations of University policy and will be pursued by the appropriate campus administrator. Finally, my signature below signifies that the work included is my own, and that I completed this assignment honestly.

Signature: _____

Sanctions for academic dishonesty include suspension or dismissal from the university. There are alternatives to academic dishonesty. Please see you professor, advisor, or the Dean of Students to discuss other choices.

Point distribution:

Short Answers: 200 points _____

Long Problem: 300 points _____

Proof: 25 points extra credit _____

Total _____

And best of luck!

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I. Short Answer (200 points. 15 questions)

1. Given the production function $q = AL^\alpha K^{(1-\alpha)}$

where L = labor employed in producing q

K = capital employed in producing q

Find the elasticity of production with respect to Labor,

2. For a square $N \times N$ matrix, A , which of the following statements is NOT equivalent to the others:

a) A is invertible

b) Every system $A\mathbf{x} = \mathbf{b}$ has at LEAST one solution for every \mathbf{b} . (\mathbf{x} and \mathbf{b} are N dimensional vectors.)

c) Every system $A\mathbf{x} = \mathbf{b}$ has at MOST one solution for every \mathbf{b} . (\mathbf{x} and \mathbf{b} are N dimensional vectors.)

d) A is singular.

e) A has maximal rank of N .

3. What is the length of the vector $\mathbf{v} = (1, 1, 1, 1)$?

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4. Which of the following sets of vectors is a BASIS of \mathbb{R}^3 ?

A) $(1, 1, 1)$, $(0, 0, 0)$ and $(1, 0, 0)$

B) $(1, 0, 0)$, $(0, 1, 0)$ and $(3, 7, 13)$

C) $(1, 0, 0)$, $(2, 0, 0)$ and $(3, 0, 0)$

D) $(1, 0)$, $(0, 1)$ and $(1, 1)$

E) $(1, 0, 0, 0)$, $(0, 1, 0, 0)$, $(0, 0, 1, 0)$ and $(0, 0, 0, 1)$

5. Does the point $(2, 4, 4)$ lie on the plane $(x, y, z) = (1, 2, 3) + t(1, 1, 0) + s(0, 1, 1)$?

6. Find the first derivative of the following function:

$$y = (x^2 + 3x + 1)^5$$

7. How long does it take an economy to double in size if it is growing at the continuous rate of 3.5% per year?

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8. Given the quadratic equation: $x^2 + y^2 + z^2 = 0$

There exists some symmetric matrix, A , and vector $\mathbf{x}^T = (x, y, z)$ such that

$$\mathbf{x}^T \mathbf{A} \mathbf{x} = x^2 + y^2 + z^2 = 0$$

Find the matrix A for which the above equation is true?

9. Find the first and second derivatives for the function $f(x) = (\ln x)^2$

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10. Solve the following constrained optimization (find x^* , y^* that maximize $f(x,y)$):

Maximize $f(x, y) = \ln(x) + \ln(y)$

subject to the constraint that $x + 2y = 3$

Extra Credit: What is the value of λ ?

11. What is the Hessian (matrix of second partials) for the following function?

$$f(x, y) = \ln(x) + \ln(y)$$

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12. The economy of Mason Land exports only Wizmoes. The quantity of Wizmoes exported, X , is increasing by 4% each year. The price of Wizmoes, P , is growing by 2% each year. What is the annual rate of growth of export revenue? (Hint – Export Revenue = $P * Q$)

A) 4%

B) 2%

C) 6%

D) 8%

E) Cannot determine without knowing the total GDP of Mason Land.

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Consider the Wizmo industry. Each firm faces the the Total Cost function for the production of Wizmoes:

$$TC = q^3 - 10q^2 + 100q$$

13. What is the quantity of Wizmoes, q , which minimizes a single firm's per unit (average total) cost. (I.e. what is the most efficient scale of production?)

14. If there is free entry and exit in the Wizmo industry, what is the long run market price of Wizmoes?

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15) For the following system of equations:

$$\begin{aligned}x + y + z &= 3 \\-x + y + 2z &= 2 \\2x - 3y + z &= 0\end{aligned}$$

Set up the system in matrix form, $Ax = b$, and show that the matrix A is non-singular.

Find the values of (x, y, z) which solve for $b = (3, 2, 0)$

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Part II: Pick three of the following four questions. DO NOT DO ALL FOUR QUESTIONS. IF YOU DO FOUR QUESTIONS I WILL ONLY GIVE CREDIT FOR YOUR THREE LOWEST!!!!

Long Problem 1

“E-Gadds”

Gadd Inc. (now the 5th largest producer of gadgets in Ohio) adopts a new production process on the line at their plant in Akron. As the employees learn to handle the new process, production per hour rises at a rate of increase of 2% per hour for every hour in operation. (Assume all productivity growth is continuous)

A. Assuming Gadd Inc started with an output of 100 gadgets/hour at 9 am on Monday morning, April 8, 2002, what do you predict will be the output 40 hours later, at closing time on Friday, April 12, 2002? (Assume productivity compounds continuously.)

B. How many hours will it take production to double?

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C. Gadd Inc's competitor, Acme Gadget, is also improving their production process. If Acme starts the week with an output of 90 gadgets per hour and ends the week (40 work-hours later) with an output of 134 gadgets per hour, what is the average percentage increase per hour in productivity per hour?

D. (Something completely different, but I needed to test another concept.)
You own a fine cheese whose value is given by the formula:

$$C(t) = (3/2)^{t^{(1/2)}}$$

Where t is in years. If the interest rate is currently 10%. When should you sell your cheese.

(Assume that today, at the starting point, $t = 0$)

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Long Problem 2

Consider the problem facing a revenue maximizing kleptocrat¹ choosing the level of taxes on the sale of Gadgets.

Assume the long run supply curve for Gadgets is infinitely elastic at a price of \$1 per gadget. (That is, the long run marginal cost of gadgets, $d(\text{Cost})/dq = 1$) This is an important assumption, because it means in the long run the supply curve is FLAT, the price that firms take home is always driven to \$1.00 per unit.

Assume the Demand curve for gadgets is given by:

$$Q_d = 100 - 10 P$$

A) What is the long run market equilibrium price and quantity in the ABSENCE of taxes.

B) What is the long run market equilibrium if the kleptocrat imposes a tax of \$1.00 on each Gadget. (The equilibrium values you must find are: the price paid by consumers, the price received by firms, the tax per unit going to the government, the total number of units traded, and the total revenue collected by the government.)

¹Kleptocrat = a government official who uses the power of the government to maximize the total “profit” (revenue – costs) of government.

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C) What is the tax that MAXIMIZES the revenue raised by the government? What is the MAXIMUM REVENUE the government takes home and what is the total quantity traded in the market at this tax rate?

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Long Problem 3

A firm has a production function $q(K,L) = A K^\alpha L^{1-\alpha}$ and unit capital and labor costs of r and w , respectively. The firm can spend no more than C per week on inputs L and K .

A) Find K^* and L^* which maximize q for fixed C , expressing K^* and L^* in terms of w , r , C and α .

B) Find the maximum weekly output achievable, q^* , produced by K^* and L^* .
(Express q as a function of w , r , C , α , and A , but NOT K or L)

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Long Problem #4

A) A factory makes 2 goods, grommets and widgets.

To make \$1 worth of grommets requires \$0.20 worth of grommets and \$0.1 worth of widgets and 1 worker/hour.

To make \$1 worth of widgets requires requires \$0.05 worth of grommets and \$0.1 worth of widgets and 0.5 worker/hours.

Set up the linear system of Equations that describes this economy:

Consumption Grommets C_G =

Consumption Widgets C_W =

Labor Employed L =

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B) If the firm is planning to sell \$750 worth of grommets and \$500 worth of widgets, what must be the total production of grommets, the total production of widgets, and the number of workers employed?

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Proof: extra credit

Given that $y = e^{g(x)}$ and that $d(e^x)/dx = e^x$, prove that $dy/dx = e^{g(x)} * dg(x)/dx$

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Scratch