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Make-up Quiz #1.
Material Covered. Klein 1, 2; Schaums 1, 2
1. Solve the following system of equations for x, y and z: $x/2 + y/4 + z = 4$
$(x + y)/3 = 1$
$x + y + z = 6$

This is a linear system. As long as the three equations are independent, it will have one solution set. To find it, I will solve the second equation for x, then substitute into the third equation. This will yield a little bonus – the y drops out and I get a value for z.

$(x + y)/3 = 1$ $\quad x + y = 3$ $\quad x = 3 - y$
$x + y + z = 6$ $\quad (3 - y) + y + z = 6$ $\quad 3 + z = 6$ $\quad z = 3$

Now back substitute:
$0.5x + 0.25y + z = 4$ $\quad 0.5(3 - y) + 0.25y + 3 = 4$ $\quad 1.5 - 0.5y + 0.256 = 1$
$-0.25y = -0.5$ $\quad y = 2$ $\quad x = 3 - 2 = 1$

So the solution set is (1, 2, 3)

Looking Forward – can you set this up as a linear (matrix) algebra problem?

2. Which of the following is the ECONOMISTS inverse of the function $y = 2x^{-1/3}$
(i.e. – find x as a function of y, x = f(y))

a) $x = 2/y^{1/3}$ b) $x = 8y^{-3}$ c) $x = 4y^{3}$ d) $x = 4y^{-1/3}$ e) $x = 3/(y^{1/8})$

Need to solve for x as a function of y.
$0.5y = x^{-1/3}$ $\quad 2y^{-1} = x^{1/3}$ $\quad 8y^{-3} = x$ $\quad$ *So the answer is b.*

3. Consider the following supply and demand functions in the Market for Widgets:

Demand: $Q_d = 9 - P^2$

Supply: $P = 2$

Find the equilibrium Quantity and Price.

Don't get thrown by the fact that half of your work is already done, you already know $P^* = 2$. Therefore $Q^* = 9 - 4 = 5$.

4. (Worth 4 points) Match the following four functions with their graphs:

i. $y = x^2 - 9$ ii. $y = x^2$ iii. $y = 1/x$ iv. $y = 3x + 2$

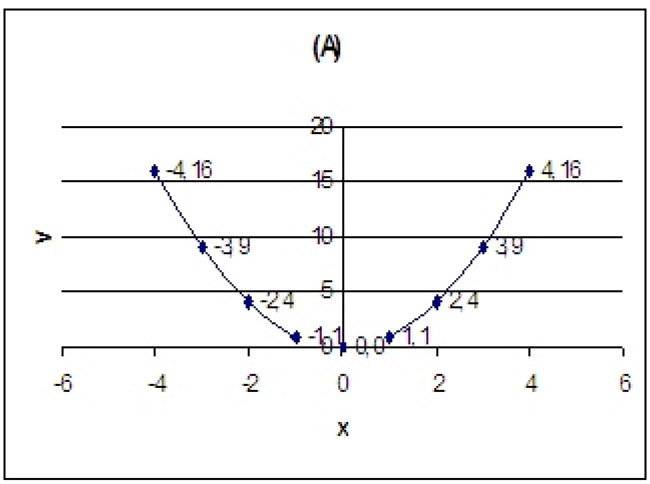
How to think about a problem like this: First – I like to solve each function for x = 0. Then look at each graph and see if the (0, y) point is on the graph:

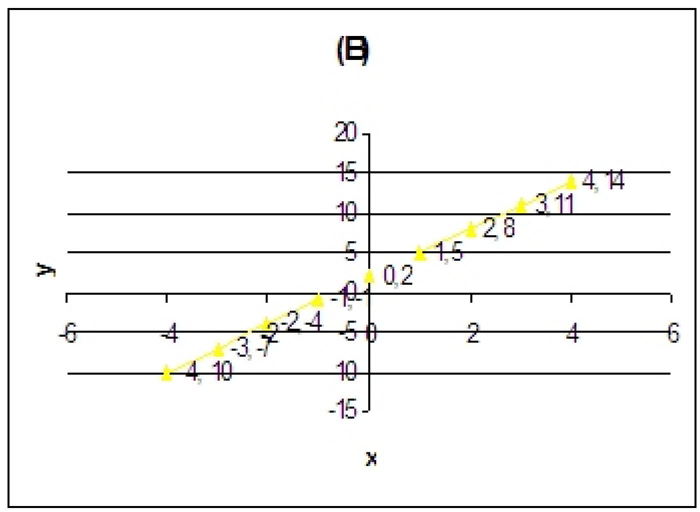
i) $y = 0^2 - 9$; *(0, – 9)* *Matches C*

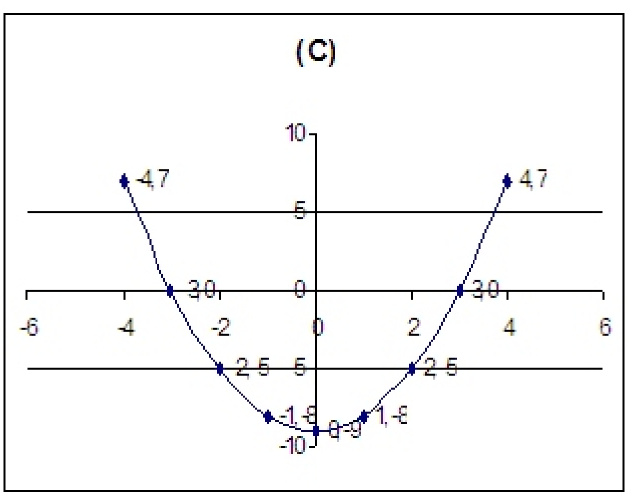
ii) $y = x^2$; *(0, 0)* *Matches A*

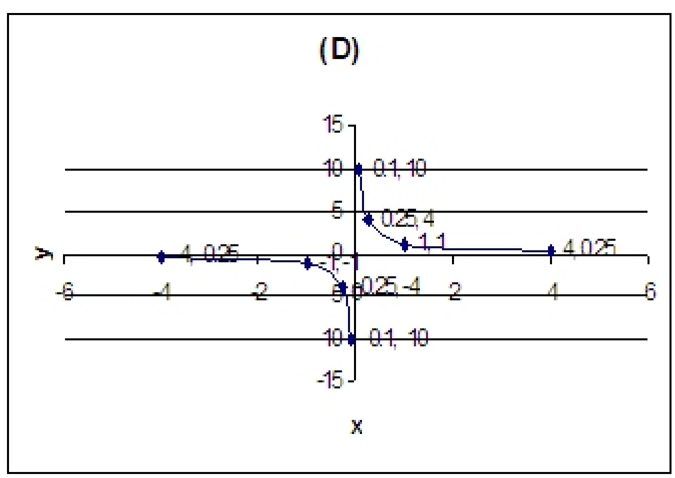
iii) $y = 1/x$ *not defined at x = 0* *Matches D*

iv) $y = 3*0 + 2$ *(0, 2)* *Matches B*









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Make-up Quiz 2: Schaum's 7, 8 (Exponents and Logs, interest compounding), Klein

1. Solve for x: $e^x = 12.5^{x+3}$

Use the trick of taking the natural log to get the x "down."

$\ln(e^x) = \ln(12.5^{x+3})$ $\quad$ $x \ln e = (x + 3) \ln(12.5)$ $\quad$ $x = (x + 3)2.53$

$x = 2.53x + 7.59$ $\quad$ $-1.53x = 7.59$ $\quad$ $x = -(7.59/1.53)$

$x = -4.96$

Checking ourselves:

$e^{-4.96} = 1/e^{4.96} = 0.0070$ $\quad$ $(12.5)^{-4.96+3} = (12.5)^{-1.96} = 1/(12.5)^{1.96} = 0.0071$

Close enough, given the rounding error.

2. Which of the following is equivalent to y if $\ln y = \ln(8) - 3\ln(x)$

a) $y = \ln(8x^3)$ $\quad$ b) $y = 8x^3$ $\quad$ c) $y = 8/x^3$ $\quad$ d) $y = e^{8x\wedge 3}$ $\quad$ e) $\ln y = 8xe^{-3}$

This involves going in the OPPOSITE direction, "un-logging"

$\ln y = \ln 8 - \ln x^3$ $\quad$ $\ln y = \ln(8/x^3)$ $\quad$ $y = 8/x^3$ $\quad$ So the answer is c)

3. $\text{Log}_2\ 64 =$

a) 1 $\quad$ b) 2 $\quad$ c) 3 $\quad$ d) 4 $\quad$ e) 5 $\quad$ f) 6

We want to find the x that solves $2^x = 64$. The answer is $2*2*2*2*2*2 = 2^6$. Answer is f)

4. You have an investment which is growing at 25% per year, compounded continuously. If the investment is worth $100 on January 1, 2004, what will it be worth on January 1, 2005.

a) 118.40 $\quad$ a) 125.00 $\quad$ b) 128.40 $\quad$ c) 148.40 $\quad$ d) 100.00

Formula: $\quad$ $V(t) = V(0)e^{rt}$

$V(1) = 100*e^{0.25} = 128.40$ $\quad$ The answer is b)

5. In 1980 an ounce of gold cost about $600, today (24 years later) a pound of gold costs about $400. What was the annual average rate of growth, compounded continously (hint – You can use the ordinary growth formula, but your answer will be < 0, because we have had negative growth = shrinkage).

We use the Value formula again: $V(t) = V(0)e^{rt}$ $\quad$ $t = 24$; $V(0) = 600$; $V(t) = 400$, we are looking for r.

$400 = 600e^{24r}$ Solution: $2/3 = e^{24r}$ $\quad$ $\ln(2/3) = 24r$ $\quad$ $r = -0.017$ or -1.7 %. The value of gold shrank by 1.7% each year.

Name

Make-up Quiz 3: Schaums 3, Klein

1. Find the first and second derivatives of $y = (5x^3 - 7x^2)^2$

There are several ways to approach this problem: a) multiply the equation out or b) use the chain rule. In this case, multiplying out is quicker, especially when we have to find the second derivative:

$$y = (5x^3 - 7x^2)^2 = 25x^6 - 70x^5 + 49x^4$$

Therefore $y' = 150x^5 - 350x^4 + 196x^3$
$y'' = 750x^4 - 1400x^3 + 588x^2$

We can also use the chain rule, which is probably easier if all we want to do is find the first derivative:

$$y = (5x^3 - 7x^2)^2$$
$$y' = 2(5x^3 - 7x^2)(15x^2 - 14x) = (10x^3 - 14x^2)(15x^2 - 14x)$$

Then use the product rule to find the second derivative:

$$y'' = (30x^2 - 28x)(15x^2 - 14x) + (10x^3 - 14x^2)(30x - 14)$$

Either answer is correct, but let's prove they are in fact the same, by multiplying out our second answer and re-arranging to simplify until we get our first answer:

$$y'' = 450x^4 - 420x^3 - 420x^3 + 392x^2 + 300x^4 - 420x^3 - 140x^3 + 196x^2$$
$$y'' = (450 + 300)x^4 - (420 + 420 + 420 + 140)x^3 + (392 + 196)x^2$$
$$y'' = 750x^4 - 1400x^3 + 588x^2$$

2. Consider the function $y = 1/x$ over the interval $-\infty < x < 0$, the negative values of x

THE KEY TO THIS QUESTION IS REMEMBERING THAT YOU ARE LOOKING AT THE BEHAVIOR OF THE FUNCTION OVER THE NEGATIVE VALUES OF X ONLY.

a) is the function continuous over this interval ($-\infty < x < 0$) (explain)?

YES. The function does not exist for $x = 0$, but $x = 0$ is not in this open interval.

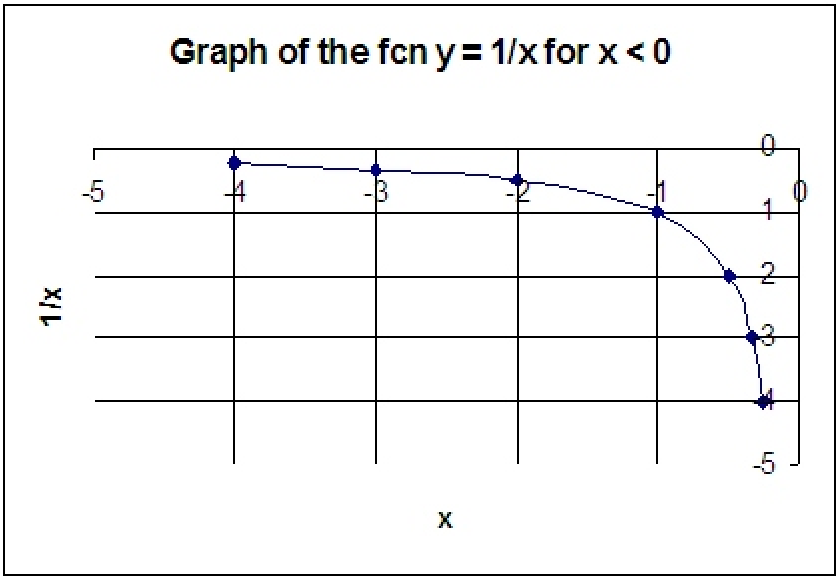
b) is the function convex or concave over the interval (explain)?

Convex functions are functions where the second derivative is POSITIVE. Concave functions are functions where the second derivative is NEGATIVE.

$y = x^{-1}$ $dy/dx = -1(x^{-2})$ $d^2y/dx^2 = 2x^{-3} = 2/x^3$

Our first instinct is to look at y" = $2/x^3$ and say "Aha, positive!", but this would be wrong because x is negative (remember $-\infty < x < 0$) When $x < 0$, y" = $2/x^3$ is also negative, therefore the function is CONCAVE.

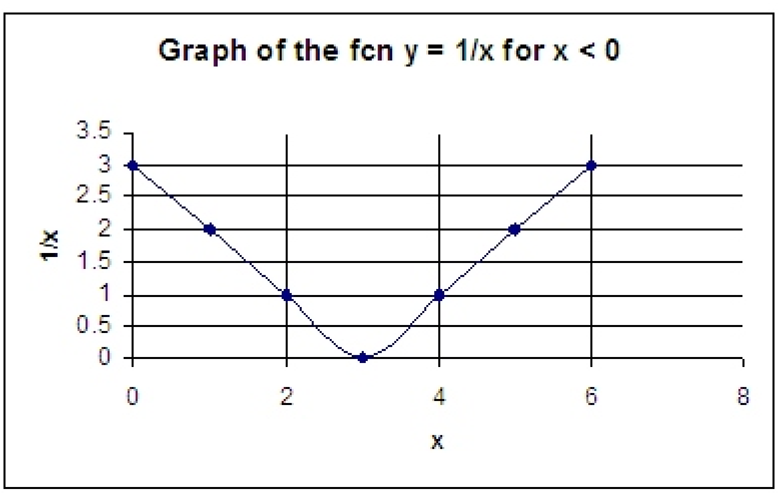
If you sketch the function you will see you have a function where the slope is becoming ever more steeply negative, which means the first derivative is falling in value, and therefore the second derivative is negative. The function must be CONCAVE. You might also note that a secant drawn between any two points on the function will lie BELOW the function, which is the official definition of concave.



3. For which values of x is the first derivative of the following functions NOT defined:

a) $y = (1/x^2)$
The first derivative is not defined if the function is not continuous, so no first derivative at $x = 0$.

b) $y = | x - 3 |$
The first derivative is not defined if the function contains a "kink" or corner, so no first derivative where this function goes from negative to positive, which happens at $x = 3$. To see this, just sketch the function:



4. Given the function $f(x) = 2x^2 - 3x$, find the first, second, third and fourth derivatives.

$y' = 4x - 3$
$y'' = 4$
$y^{iii} = 0$
$y^{iv} = 0$

REMEMBER THAT THE DERIVATIVE OF A CONSTANT = 0, AND 0 IS A CONSTANT, SO THE DERIVATIVE OF 0 = 0.

5. Given the functions $z = 13y^2$ and $y = x - 1/x$, find dz/dx

This is the chain rule: dz/dx = dz/dy * dy/dx

$dz/dy = 26y = 26(x - 1/x)$

$dy/dz = 1 + x^{-2}$

$dz/dx \quad = 26(x - 1/x)(1 + 1/x^2)$

Name:

Make-up Quiz 4: Schaum's 4, 9

1. Find the first and second derivative for the following function:

$f(x) = \ln(2x) + e^{2x}$

$df/dx = 2/2x + 2e^{2x} = x^{-1} + 2e^{2x}$ *Chain rule*

$d^2f/dx^2 = -x^{-2} + 4e^{2x}$

2. Consider the function $f(x) = (16/3)x^3 - 4x$

a) Find the critical points. Remember, a point has two coordinates (x*, f(x*))

First Order Conditions for a critical point: $f'(x) = 0$
$f'(x) = 16x^2 - 4 = 0$ $x^2 = 4/16 = 1/4$ $x = ½$
$f(½) = (16/3)(½)^3 - 4*(½) = (16/3)*(1/8) - 2 = 2/3 - 2 = -4/3$
Our critical point is (½, – 4/3)

b) For each point, determine if it is a local maximum or a local minimum.

Second Order Conditions determine if we have a min or a max. Sign on $f''(x)$
$f''(x) = 32x$ $f''(½) = 16 > 0$ We have a minimum

Check – what is happening to the first derivative around x = ½? When x = 0, f'(x) = – 4, when x = 1, f'(x) = 12, so clearly the first derivative goes from negative (downward sloping curve) to positive (upward sloping curve), so we must have a minimum between.

3. For the Total Cost Function $TC(q) = q^3 - q^2 + 100q$, find the Average Cost and the Marginal Cost functions.
Average Cost = Total Cost ÷ q = $q^2 - q + 100$
Marginal Cost = $d(TC)/dq = 3q^2 - 2q + 100$

4. For the Total Cost function question 3, find the **minimum average cost** and the level of q at which this minimum average cost is achieved.

2 ways to do this – a) find the critical point(s) for the AC function, and determine if one is a minimum, or b) find the q for which AC = MC.

Method a) $d(AC)/dq = 2q - 1 = 0$ when q = ½, AC(½) = (1/4) – ½ + 100 = 99.75
Method b) $q^2 - q + 100 = 3q^2 - 2q + 100$ $q^2 - q = 3q^2 - 2q$ $q - 1 = 3q - 2$
$q - 1 - q + 2 = 3q - 2 - q + 2$ $1 = 2q$ $q = ½$
Yup, same ansnwer. And therefore AC(½) = min AC = 99.75

5. A monopolist which sells good x has the profit function:

$\pi(x) = (10 - x)x - 2x$ Where x = quantity produced

Find the profit maximizing output, x_M, and the monopoly profit, $\pi(x_M)$

max $\pi(x) = \max_x (10 - x)x - 2x = \max_x 10x - x^2 - 2x = 8x - x^2$
First Order Conditions require first drv, $d\pi/dx = 0$

$d\pi/dx = 8 - 2x = 0$ *when* $x = 4$, *So monopoly profit* $= 8*4 - 4*4 = 32 - 16 = 16$. *Our answer is:*
$x^M = 4$, $\pi^M = 16$.

Attentive readers may notice that the structure of our profit function suggests Total Revenue = (10 – x)x and Total Costs = 2x. So we have a constant returns to scale production function where AC = MC = 2, and a Demand Function that looks like P = (10 – x)

Name:

Make-up Quiz 5: 3, 7, and 9 (Review Quiz)

Consider the following Demand and Supply functions:

Demand: $Q = 1/(2P) = (2P)^{-1}$ for $P > 0$
Supply $P = MC = AC = 0.25$

1. Find the market clearing Price and Quanity, P* and Q*.

P = P = 0.25 is given by the flat supply curve. So Q* = 1/(2*0.25) = 1/.5 = 2*

2. Find the Price Elasticity of Demand as a function of P.
Hint: The formula for elasticity is $\epsilon = dQ/dP * (P/Q)$

Just fill in the formula: $dQ/dP = -0.5 P^{-2}$ $Q = 0.5 P^{-1}$

$\epsilon = [-0.5 P^{-2}]*P/0.5 P^{-1} = P^{2} P^{-2} = -1$

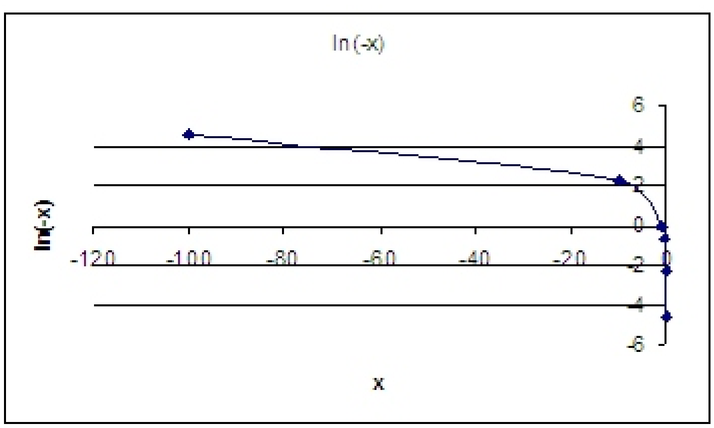
3. Is the demand function $Q = 1/(2P) = (2P)^{-1}$ convex or concave in P? Explain.

The second derivative will tell us: $dQ/dP = -0.5P^{-2}$ $d^2Q/dP^2 = P^{-3} = 1/P^3$

Which means for positive Prices the second derivative is Positive and the demand function is convex (for negative prices it will be concave, but negative prices are rare.) Do note that the graph is very misleading, because on the supply/demand diagram the independent and dependent axes are flipped.

4. What is the domain and what is the range of the function $f(x) = \ln(-x)$

Can't take the ln of a negative number, so $(-x) > 0$ *for f(x) to be defined, therefore the domain is x is negative,* $-\infty < x < 0$*. The range, the values of y, is* $(-\infty, \infty)$*. See the graph below:*



5. On January 1, 2004 Alan invests \$100.00 at 5% interest per year, Betty invests \$50.00 at 9% interest per year, both interest rates are compounded continuously. In how many years will the value of Betty's investment = the value of Alan's investment?

Plan: Set up the formulae for Alan's investment and Betty's investment as functions of t. Set them equal to each other and solve for t.

Alan: $V_A(t) = 100e^{0.05t}$ *Betty:* $V_B(t) = 50e^{0.09t}$

So now we need to find the value of t at which these two formulae solve to the same value.

$$V_A(t) = V_B(t)$$

$$100e^{0.05t} = 50e^{0.09t}$$
$$2e^{0.05t} = e^{0.09t}$$
$$\ln 2 + 0.05t = 0.09t$$
$$\ln 2 = t(0.09 - 0.05) = 0.04t$$
$$t \approx 0.69/0.04 = 17.25$$

Name

Make-up Quiz 6: Schaum's 4, 8 and 9 (a review of economic applications)

1. Find the first and second derivative for the function $y = 7^{1/3}x^{2/3}$

$dy/dx = (7^{1/3}) * (2/3) * x^{-1/3} \approx 1.275x^{-1/3}$

$d^2y/dx^2 = (7^{1/3}) * (2/3) * (-1/3)\, x^{-4/3} = -0.4251x^{-4/3}$

2. When Tommy entered college his parents promised him 20,000 when he graduated. Tommy can finish college in 4 years, or he can relax (take fewer classes) and finish in 5 years. If the current interest rate (compounded continuously) is 5% per year, how does the PRESENT value of the promised 20,000 change if Tommy graduates in 5 years instead of 4?

Present Value formula: $V(today) = V(t)\, e^{-rt}$
If $t = 4$ years: $V(today) = 20{,}000 * e^{-0.05 * 4} = 20{,}000 * e^{-0.2} = 20{,}000 * .8187 = 16374.62$
If $t = 5$ years: $V(today) = 20{,}000 * e^{-0.05 * 5} = 20{,}000 * e^{-0.25} = 20{,}000 * .7788 = 15576.02$

The difference in present value is 16,374.62 – 15,576.02 = $798.60

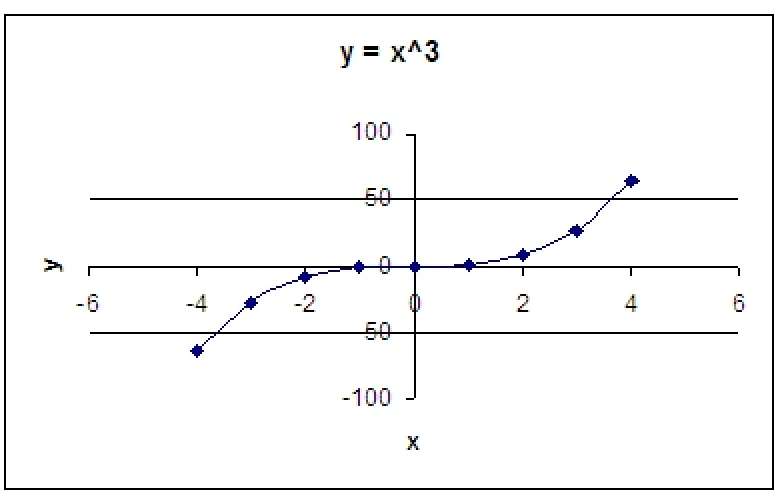
3. (4 points) Find the critical point(s) and determine if it or they are minima, maxima, or neither. Explain your answer.

a) $f(x) = x^2$

$f'(x) = 2x$ $\quad\quad$ $f''(x) = 2$

The critical point is (0,0) and it is a minimum (second drv > 0)

b) $g(x) = x^3$ (If the second derivative doesn't give the answer, try sketching the function.)
$f'(x) = 3x^2$ $\quad$ $f''(x) = 6x$



Answer:
Critical point is again (0,0). The second derivative is also equal to zero, but if we sketch the function around (0,0) we see that we have a point of inflection, neither minimum nor maximum.

4. If $y = \ln(x^2)$ and $z = 2y$, what is dz/dx?

Chain rule: $dz/dx = dz/dy * dy/dx = 2 * 2x/x^2 = 4/x$

Check – multiply out to get $z = 2(\ln(x^2)) = \ln(x^4) = 4x^3 / x^4 = 4/x$

Name:

Make-up Quiz 7: Schaums' Chapters 5 and 6: (Calculus of Multivariable Functions)

1. (4 points) Given the function: $z = e^{xy} - \ln(xy)$

a) Find the two first partial derivatives: $\partial z/\partial x = y\, e^{xy} - 1/x$ $\partial z/\partial y = x\, e^{xy} - 1/y$

b) Find the four second partial derivatives $\partial^2 z/\partial x^2 = y^2\, e^{xy} + 1/x^2$ $\partial^2 z/\partial x\partial y = xy\, e^{xy}$

$\partial^2 z/\partial y\partial x = xy\, e^{xy}$ $\partial^2 z/\partial y^2 = x^2\, e^{xy} + 1/y^2$

2. Given the Production Function $Q = 25K^{0.4}L^{0.6}$

Find the Marginal Product of Labor and the Marginal Product of Capital.

Marginal Product of Labor $= \partial Q/\partial L = 25*0.6\ (K/L)^{0.4}$

Marginal Product of Capital $= \partial Q/\partial K = 25*0.4\ (L/K)^{0.6}$

3. Given the profit function $\pi = 25x - x^2 - xy - 2y^2 + 30y - 28$

Find the quantities of x and y that maximize profits.

Need to find the x and y that solve for the first order conditions:

$\partial\pi/\partial x = 25 - 2x - y = 0$ $\partial\pi/\partial y = -x - 4y + 30 = 0$

$y = 25 - 2x$ $-x - 4(25 - 2x) + 30 = 0$

$-x - 100 + 8x + 30 = 0$

$7x - 70 = 0$ $x = 10$

$y = 25 - 20 = 5$ $y = 5$

Check this is a MAX, not a min $\pi_{xx} = -2 < 0$ $\pi_{xy} = -1$

$\pi_{yx} = -1$ $\pi_{yy} = -4 < 0$

And $\pi_{xx}*\pi_{yy} - (\pi_{xy})^2 = 8 - 1 = 7 > 0$. *Okay, we have a max.*

4. Find the limit of function $z = e^{-xy}$ as x and y go to infinity: $\lim_{x, y \to \infty} 1/e^{xy} = 0$

If x and y are both going to infinity, then their product, xy, is going to infinity, and the limit of $1/\infty$ *is ZERO.*

Name:

Make-up Quiz 8: Schaum's 5 and 6 (Especially 6, focus on applications)

1. (4 points) A monopolist sells two products, x and y, with the inverse demand functions:

$p_x = 10 – x$

$p_y = 4 – y/3$

and the combined cost function is $c = x^2 + xy + y^2 + 10$

a) What is the monopolist's Profit function?

π = *Total Revenue – Total Profits*
$= p_x * x + p_y * y – x^2 – xy – y^2 – 10$
$= (10 – x) * x + (4 – y/3) * y – x^2 – xy – y^2 – 10$
$= 10x – x^2 + 4y – y^2/3 – x^2 – xy – y^2 – 10$
$= 10x – 2x^2 + 4y – 4y^2/3 – xy – 10$

b) What are the values of x and y which maximize profits.

Plan – set up the first order conditions, solve for x and y.
first order conditions for a max: $\partial\pi/\partial x = 10 – 4x – y = 0$
$\partial\pi/\partial y = 4 – 8y/3 – x = 0$
Solving for x and y:
$y = 10 – 4x$ $x = 4 – 8y/3$ $3x = 12 – 8y$

$3x = 12 – 8(10 – 4x) = 12 – 80 + 32x = 32x – 68$

$29x = 68$ $x = 68/29 \approx 2.34$ $y = 10 – 4*2.34 = 0.62$

2. Given the production function $q = 2L^2K + 3K$, what is the Labor elasticity of output.

Remember, the elasticity of a function f(x,y) with respect to x = $\epsilon_{y,x} = \partial f/\partial x * (x/f(x))$

$\epsilon_{a,L} = \partial q/\partial L * (L/q) = 4LK * L/(2L^2K + 3K) = (4L^2) / (2L^2 + 3)$

3. Which of the following functions exhibits constant returns to scale (doubling all inputs exactly doubles outputs)

a) $Q = 10KL$ b) $Q = 10K^{1/2}L^{1/2}$ c) $Q = 2K + 3L$ d) $Q = 2L^2K + 3K$

Answer: b and c both work.

Consider the standard equation for constant percent growth:

$V(P,r,t) = Pe^{rt}$ Where V = Value at t years in the future
P = Principal invested today
r = percent rate of growth

4. Find the three first partials:

$\partial V/\partial P =$ e^{rt}

$\partial V/\partial r =$ r

$\partial V/\partial t =$ t

5. Why can't you find $\partial V/\partial e$?

Because e is a number, $e \approx 2.71$. Can't take derivative with respect to something that never changes, like a number.

Name

Make-up Quiz 9: Schaum's 5 and 6 Optimization

1. Consider the function $z = x^2 + xy - 3y$

Find the critical points for this function and identify them as minima or maxima or saddles or cannot determine.

$\partial z/\partial x = 2x + y = 0$
$\partial z/\partial y = x - 3 = 0$ $\quad x = 3 \quad y = -6 \quad$ this is the only critical point

$z_{xx} = 2 \quad z_{xy} = 1$
$z_{yx} = 1 \quad z_{yy} = 0$

We cannot determine from the second order conditions if this is a min or a max or a saddle, because $z_{yy} = 0$.

2. For which of the following functions is (x, y, z) = (0, 0, 17) a critical point?

a) $z = x + y + 17$ b) $z = x^2 + y^2 + 17$ c) $z = \ln(17x^2y^2)$

*First we can eliminate c), because ln(17*0*0) is not equal to 17.*
Then we can eliminate a) because it is a linear function and does not have any critical points. The two first partial derivatives are both constants, $\partial z/\partial x = 1$ and $\partial z/\partial y = 1$.
Therefore the only possible answer is b). We confirm this by checking the first partials: $\partial z/\partial x = 2x = 0$ when $x = 0$; $\partial z/\partial y = 2y = 0$ when $y = 0$, and $z(0,0) = 17$.

3. Given the demand function for cookies:

$Q_C = 10 - P_C - 0.5P_M$ Where Q_C = quantity of cookies
P_C = price of cookies
P_M = price of milk

What is the elasticity of demand for cookies with respect to the price of cookies and the elasticity of demand for cookies with respect to the price of milk?

Elasticity of demand for cookies wrt price of cookies:
$\epsilon = dQ_C/dP_C * (P_C / Q_C) = -P_C / (10 - P_C - 0.5P_M)$

Elasticity of demand for cookies wrt price of Milk:
$\epsilon = dQ_C / dP_M * (P_M / Q_C) = -0.5P_M / (10 - P_C - 0.5P_M)$

4. Consider the function $z = exy - e^x - ey$

Find the critical point(s) for this function and identify them as minima or maxima or saddles or cannot determine.

NOTE – exy and ey are not typos. Remember e is a number, approximately equal to 2.71

first order conditions:

$z_x = ey - e^x = 0$

$z_y = ex - e = 0$ $\quad$ $e(x - 1) = 0$ $\quad$ *therefore* $x = 1$

$ey - e^1 = 0$ $\quad$ $e(y - 1) = 0$ $\quad$ *therefore* $y = 1$

second order conditions

$z_{xx} = -e^x$ $\quad$ $z_{xy} = e$

$z_{yx} = e$ $\quad$ $z_{yy} = 0$

Can't determine if this is a min, max or saddle because $z_{yy} = 0$.

Name

Make-up Quiz 10: Schaum's Chapters 5 and 6 – constrained optimization

1. Constrained optimization – a firm manufactures two goods, x and y. The total cost function is:

$$TC = x^2 + 4y^2 - 2xy + 100$$

The firm has a contract to produce 10 units of output: $x + y = 10$

Using a Lagrangian, find the quantities of x* and y* which will minimize production costs. What is the value of the "shadow price" λ* at x* and y*.

$\mathcal{L} = (x^2 + 4y^2 - 2xy + 100) + \lambda(10 - x - y)$

f.o.c.:

$\partial\mathcal{L} / \partial x = 2x - 2y - \lambda = 0$

$\partial\mathcal{L} / \partial y = 8y - 2x - \lambda = 0$

$\partial\mathcal{L} / \partial \lambda = 10 - x - y = 0$

$2x - 2y = \lambda = 8y - 2x$ $\quad x - y = 4y - x$ $\quad 2x = 5y$ $\quad x = (5y)/2$

$10 - (5y)/2 - y = 0$ $\quad 20 - 5y - 2y = 0$ $\quad 20 = 7y$ $\quad y = 20/7$

$x = (5/2)(20/7) = 100/14 = 50/7$

$\lambda = 2x + 2y = 100/7 + 40/7 = 140/7 = 20$

$x^* = 50/7$

$y^* = 20/7$

$\lambda^* = 20$

2. Two inputs are used to produce coffee, capital (coffee makers, cups, etc.) and labor. The production function for coffee is: $q = 12\ K^{1/4}\ L^{3/4}$

Where K = quantity of capital required
L = quantity of Labor required, and
q = number of cups of coffee produced per hour
The current WAGE (price of labor) is w = \$1.00 per hour.
The current RENTAL rate for one unit of capital is v = \$0.50 per hour.

The coffee shop owner wants MINIMIZE total COSTS while keeping her TOTAL OUTPUT at $q \geq 192$ cups per hour.

A. Find the "objective function" and the "constraint" and set up the Lagrangian:

Objective Function: $TC = wL + wK$
Constraint: $192 = 12\ K^{1/4}\ L^{3/4}$
Lagrangian: $\mathfrak{L} = wL + vK + \lambda(192 - 12K^{1/4}L^{3/4})$

B. Find the quantities of K and L and that maximize output, subject to the constraint.

$\partial\mathfrak{L}/\partial L = w - \lambda 12(K/L)^{1/4} = 0 \qquad w = \lambda 12(K/L)^{1/4}$
$\partial\mathfrak{L}/\partial K = v - \lambda 12(L/K)^{3/4} = 0 \qquad v = \lambda 12(L/K)^{3/4}$
$\partial\mathfrak{L}/\partial\lambda = 192 - 12K^{1/4}L^{3/4} = 0$

$w/v = [\ \lambda 12(K/L)^{1/4}\ ]\ /\ [\ \lambda 12(L/K)^{3/4}\ ] = (K/L)^{1/4}(K/L)^{3/4} = (K/L)$

$w = 1,\ v = 0.5 \qquad 1/0.5 = 2 = K/L \qquad K = 2L$
$192 = 12(2L)^{1/4}(L)^{3/4} = (12*2^{1/4})L \qquad L = 192/(12*2^{1/4}) = 13.45$
$K = 2L = 26.90$

3. If Total Revenue and Total Costs are simply functions of output, q, do you need to use a Lagrangian to solve for the maximum profit? (Explain)

No – generally maximizing the profit function is an UNCONSTRAINED maximization.