Name:_____ Econ 340, Practice Final

e-mail address:_____

Phone number:_____

General Instructions:

READ EACH QUESTION CAREFULLY. NO PARTIAL CREDIT IF YOUR ANSWER IS WRONG BECAUSE YOU MISREAD THE QUESTION.

PUT YOUR NAME ON ALL SHEETS OF THE EXAM BOOKLET, INCLUDING GRAPHS AND SCRATCH PAPER.

CALCULATORS, SLIDE RULES, PENS, PENCILS, ERASERS, AND RULERS ARE THE ONLY TOOLS YOU MAY USE ON THIS EXAM.

PLEASE READ AND SIGN THE FOLLOWING:

I understand that academic integrity is highly valued at GMU. Further, I understand that academic dishonesty, such as cheating and plagiarism, are violations of University policy and will be pursued by the appropriate campus administrator. Finally, my signature below signifies that the work included is my own, and that I completed this assignment honestly.

Signature:_____

Sanctions for academic dishonesty include suspension or dismissal from the university. There are alternatives to cheating. Please see your professor, advisor, or the Dean of Students to discuss other choices.

Best of luck!

Point Distribution: 500 points total
Short Answers: 300 points
Long Problem: 200 points

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Short Answers (15 questions, 20 points each)

1. Which of the following statements are true about the following system of equations? (There may be more than one right answer; you must evaluate EACH statement to get full credit.)

$$3x - 7y + 2z = 120$$
$$-x + y + z = 10$$
$$2x - 6y + 3z = 100$$

a) The system of equations has no solution
b) The system of equations has a single, unique solution set
c) The system of equations has an infinite number of solution sets
d) The determinant of the matrix of coefficients is zero.
e) The three equations are DEPENDENT (NOT independent).

2. Macy's is having a sale:

All clothes are 30% off the ticket price.

All housewares are 45% off the ticket price.

And you have a coupon for 10% off of your total purchase.

If you buy 1 shirt with a ticket price of 15.00, 2 pairs of jeans each with a ticket price of 20.00, and 1 toaster with a ticket price of 50.00, and you use your coupon, how much will you spend (before taxes)?

3. The economist's inverse of the function $y = e^{(3x + 1)}$ is:

a) $\ln y = 3x + 1$

b) $x = \frac{(\ln(y) - 1)}{3}$

c) $3x + 1 = e^y$

d) $\ln x = 3y + 1$

e) This function does not have an inverse.

4. What is the Domain of the function $y = 1/(x+2)$

5. Which of the following functions is convex (concave up) over its entire domain?

a) $y = 3x^2$

b) $y = -3x^2$

c) $y = 3x^3$

d) $y = -3x^3$

e) $y = 1/x$

6. Given the function $y = 4x^{1/2}$, which of the following is the SECOND DERIVATIVE of this function:

a) $-(1/x^{3/2})$

b) $2/x^{1/2}$

c) $2x^{1/2}$

d) $-(x^{1/2})^3$

e) $+(x^{1/2})^3$

7. Evaluate: $(64)^{5/6}$

8. $e^k =$

a) $\lim_{n \to \infty} (k/n)^n$

b) $\lim_{n \to 0} (1 + k/n)^k$

c) $\lim_{n \to \infty} (1 + k/n)^k$

d) $\lim_{n \to 0} (1 + k/n)^n$

e) $\lim_{n \to \infty} (1 + k/n)^n$

9. An economy has a labor force growing at a rate of 2% per year and each worker's productivity is growing at 3% per year, thanks to improvements in technology. How fast is annual OUTPUT growing? (Approximately – you can use either the continuous or the discrete growth model and you will get answers very, very close together.)

a) 2% b) 3% c) 4% d) 5% e) 6%

10. Find the SECOND derivative of e^{5x}.

The following two questions use this formula:

A biologist determines that under ideal conditions oak trees grow according to the following formula:

$$H = 3.5^{(\sqrt{t})}$$

Where H = Height, in inches
t = number of years since the acorn (oak seed) was planted

11. What is the height, in feet and inches, after 25 years. (1 foot = 12 inches)

12. After 25 years how quickly is the tree growing, that is what is the annual RATE of growth, in percents?

13. Assuming you can borrow and lend at a 7% interest rate, compounded continuously.

What is the Net Present Value of being paid $100.00 in one year and 200.00 in two years, and 300.00 in three years. (Total payments = 600.00 face value)

14. The determinant of the following matrix:

$$\begin{matrix} 1 & 2 & -99 \\ 0 & 3 & 27 \\ 0 & 0 & 4 \end{matrix}$$

a) 12 b) 64152 c) – 64152 c) cannot be determined d) does not exist

15. Which of the following is the product of the multiplication of:
$(1, 0, 2) \cdot (2, 0, 3)^T$?

a) (2, 0, 6) b) $(2, 0, 6)^T$ c) 8 d) (½, 0, 2/3)

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Part II: Long Problem (200 points)

Problem 1:
Answer the following questions based on the function:

$$y = 2x^2 + 8x + 3$$

A. What is the first derivative of this function?

B. What is the minimum value attained by the function (the minimum y)?

C. At what value of x is the minimum value of y achieved?

D. Over which of the following intervals is the function a DECREASING function of x?

a) $(-\infty, -2)$ b) $(\infty, 2)$ c) $(-2, 2)$ d) $(-2, 27)$ e) $(-\infty, \infty)$

$$y = 2x^2 + 8x + 3$$

E. What is the Second Derivative of this function?

F. Is this function C^{∞}. That is, can it be infinitely differentiated?

G. What is the formula for ELASTICITY for this function y with respect to x? (Don't just give me the formula for general elasticity, but rather the elasticity for $y = 2x^2 + 8x + 3$.)

$$y = 2x^2 + 8x + 3$$

H. If x is itself a function, x = 1/(t), which of the following is the composite function $(y \circ x) = y(x(t))$.

a) $3t^2 + 8t + 2$

b) $2t^2 + 8t + 3$

c) $(2t^2 + 8t + 3)/t$

d) $(3t^2 + 8t + 2)/t$

e) $(3t^2 + 8t + 2)/t^2$

I. Find dy/dt, the first derivative of the composite function, $(y \circ x) = y(x(t))$, with respect to t. (You may use the Chain Rule or your answer to part G)

J. Evaluate the function $(y \circ x) = y(x(t))$ at t = 0.25.

Problem 2: "Inspector Gadget"

Gad-Inc (now the largest producer of gadgets in Ohio) still has the production function:

$$Q = 200\ (K)^{0.25}\ (L)^{0.75}$$

where Q = gadgets per hour
L is the number of workers per hour
K is the quantity of capital employed per hour (measured in machines)

A. If Gad-Inc expands from 100 workers and 10 machines to 150 workers and 15 machines what happens to output?

a) Increases by 50% b) Increases by 100% c) Increases by 150%

d) Decreases by 50% e) Cannot be determined.

B. What are the formulas for the Marginal Physical Product of Labor and the Marginal Physical Product of Capital at Gad-Inc?

C. In a competitive economy the WAGE is the MARGINAL REVENUE PRODUCT of LABOR, MRP(L), which is itself defined as:

MRP(L) = Marginal Physical Product of Labor * Price of Output

If Gad-Inc has 81 workers, 16 machines, and Gadgets are selling for $0.33 (33 cents) each, wholesale (that is – the price that Gad-Inc is paid), what is the competitive wage rate?

Hint – you will need your answer from part B.

D. For any multivariable function $y = f(x_1, x_2, ..., x_n)$ elasticity of y with respect to x_i, or the x_i Elasticity of y, is defined as:

$$\epsilon_{y, x} = (\partial y/\partial x_i)(x_i/y) = (\partial y/\partial x_i)(x_i/f(x_1, x_2, ..., x_n))$$

Using this formula, find the elasticity of Output, Q, with respect to Labor, L, as a function of K and L only, using Gad-Inc's production function. (That is, if Q shows up in your formula, substitute.)

Hint – the Cobb-Douglas Production Function is a Constant Elasticity Function, so don't be surprised if your result is VERY SIMPLE.

E. Cheltenham Royal Widgets, Ltd. (with manufacturing facilities in Cheltenham, UK) has the cost function:

Total Costs = $Q^2 + Q + 100$

Where Q = the number of widgets

Find the Average Total Cost and the Marginal Cost of Widgets.

F. For Cheltenham Royal Widgets, find the MINIMUM Average Total Cost.

G. You are considering buying stock in Cheltenham Widget. The stock is expected to pay a dividend of $10.00 every year forever. If the current (and expected future) CONTINUOUSLY compounded interest rate is 5% what is the fair market price of stock in Cheltenham Widget?

H. You have a budget of $100.00 to spend on Widgets and Gadgets. Gadgets cost $20.00 each and Widgets cost $10.00 each. In (Gadget, Widget) space, which of the following is the equation for your BUDGET line. (Gadgets, G, are on the y-axis and Widgets, W, on the x-axis.)

a) $G = -(\frac{1}{2})W + 5$

b) $G = -2W + 5$

c) $G = \frac{1}{2}W + 5$

d) $G = \frac{1}{2}W - 5$

e) $G = 2W - 5$

End of Exam – Congratulations and have a great summer.

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Answer Key:

1. Answer: a), d) and e) are true, b) and c) are false.

Concept – if the three equations are independent then only b) will be true. If the equations are dependent then the determinant will be zero and either a) or c) might be true.

The determinant = 3(3+6) – 7(– 3 – 2) + 2(6 – 2) = 27 – 35 + 8 = 0. Therefore d) and e) are true, and b) is false. We therefore have either no solutions or an infinite number of solutions.

To determine whether a or c are true, simply pick value for x and try to solve (if there are infinite solution sets, we can pick any x and solve for the y and z that go with it.) I picked x = 10. Then solving the first two equations simultaneously yields y = 10 and z = 10, but these do not fit in the third equation.

Similarly, you can see that the third equation = the first equation + the second equation, but the solutions of the first two equations, 120 + 10, do not equal the third equation. $130 \neq 100$.

Test your understanding: How many solution sets would exist if the third equation were equal to 2x – 6y + 3z = 100? Answer: An infinite number, because then both the left hand side and the right hand side of the equation are equal to the sum of the first two equations.

2. Total Sale = 0.90*[0.70(15.00 + 2*20.00) + 0.55(50)] = 59.40

3. b)

Take ln of both sides, solve for x: $\ln y = (3x + 1) * \ln e = 3x + 1$

$x = (\ln y - 1)/3$

4. Domain is all the values of x for which the function is defined, so x = all the real numbers EXCEPT – 2, or $x = \{x \mid x \in \text{Real Numbers AND } x \neq -2\}$.

5. a)

A function is convex everywhere (a "smile") if the second derivative is ALWAYS positive. Only the first function has a second derivative that is always positive (in fact, $d^2(3x^2)/dy^2 = 6$. You can also determine this by sketching the functions.

6. a)

7. 32

Easiest to first find the sixth root, then take the power of 5. Always a good idea to start by thinking about 2:

$2*2*2*2*2*2 = 64$, so $64^{1/6} = 2$; $2^5 = 2*2*2*2*2 = 32$

8. d)

This is based on the definition of $e = \lim_{n \to \infty} (1 + 1/n)^n$

e^k = the growth that occurs in one year if the continuous interest rate is k. Conceptually, we divide the year up into n units, and we divide k up into n units, then we think about compounding the interest by k/n every n'th unit, then taking n to infinity, or

$\lim_{n \to \infty} (1 + k/n)^n$

9. e) 5%.

The quick and dirty answer is that growth rates add because powers add. There are a couple of ways to figure this out:

Quick and dirty – build yourself a little model of a world that starts with 100 workers, each of whom make \$1.00 worth of GDP: Total GDP =100. Now add 3 workers and make each worker produce \$1.02 worth of GDP. Total GDP will now equal $103*1.02 = 105.06 \approx 1.05$, which means growth is real, real close to 5%.

Using the discrete model:
$V(1) = V_0*(1+0.03)*(1+0.2) = V_0(1.00+0.3+0.2+0.06) = V_0(1.0506) \approx V_0(1.05)$

Using the continuous model:
$V(1) = (V_0\, e^{0.03})e^{0.02} = V_0\, e^{0.05}$ This is 5% growth exactly in the continuous model.

10. If $f(x) = e^{5x}$, then we use the chain rule: $df(g(x)/dx = df/dg * dg/dx$.

$f'(x) = 5e^{5x}$ $f''(x) = 25e^{5x}$

11. Height $= 3.5^{\sqrt{25}} = 3.5^5 \approx 525.22$ inches $\approx$ 43.77 feet

12. (This one is hard.) The growth RATE is given by the formula; $H'(t)/H(t)$ – that is, the change in growth per year over the current height. If you are having a good day, you will remember that $H'(t)/H(t) = d(\ln(H(t))/dt$:

$\ln(H(t)) = t^{1/2} * \ln(3.5)$

$d(\ln(H(t))) = 1/2*\ln(3.5)*t^{-\frac{1}{2}} \approx 1.25/(2*5) = .125$ inches per year.

13. Present Value $= V(t)*e^{-rt}$ Don't forget the negative sign.

So we need to compute three Present Value formulas and add them:

Value today $= 100*e^{-0.07} + 200*e^{-0.07*2} + 300*e^{-0.07*3} \approx 510.29$

14. a) 12. The determinant of an upper triangular matrix is the product of the diagonal elements

15.c) 8. The product of two vectors is a single number.

Long Problem Answers:
Problem 1:
Answer the following questions based on the function:

$$y = 2x^2 + 8x + 3$$

A. What is the first derivative of this function?
$dy/dx = 4x + 8$

B. What is the minimum value attained by the function (the minimum y)?

First order condition for a minimum is $dy/dx = 0$
$4x + 8 = 0$ $x = -2$, therefore $y = 2*(-2)^2 + 8*(-2) + 3 = 8 - 16 + 3 = -5$. The minimum value attained by the function is -5.

C. At what value of x is the minimum value of y achieved?
The minimum is achieved at $x = -2$.

D. Over which of the following intervals is the function a DECREASING function of x?

a) $(-\infty, -2)$ b) $(\infty, 2)$ c) $(-2, 2)$ d) $(-2, 27)$ e) $(-\infty, \infty)$

Answer – a), because the function has a negative first derivative over the interval $(-\infty, -2)$

E. What is the Second Derivative of this function?

$y'' = 4$

F. Is this function C^{∞}. That is, can it be infinitely differentiated?

Yes, although the derivatives above second are all equal to zero.

G. What is the formula for ELASTICITY for this function y with respect to x?

(Don't just give me the formula for general elasticity, but rather the elasticity for y = $2x^2 + 8x + 3$.)

Elasticity = dy/dx * (x/y) = $(4x + 3)*(x/2x^2 + 8x + 3) = (4x^2 + 3x)/2x^2 + 8x + 3)$

H. If x is itself a function, x = 1/(t), which of the following is the composite function (y ◦ x) = y(x(t)).

a) $3t^2 + 8t + 2$

b) $2t^2 + 8t + 3$

c) $(2t^2 + 8t + 3)/t$

d) $(3t^2 + 8t + 2)/t$

e) $(3t^2 + 8t + 2)/t^2$

To find the composite function, substitute in x = 1/t, which yields $y = 2t^{-2} + 8t^{-1} + 3$, which is the same as $y = (2 + 8t + 3t^2)/t^2$

Ans is e)

I. Find dy/dt, the first derivative of the composite function, (y ◦ x) = y(x(t)), with respect to t. (You may use the Chain Rule or your answer to part G)

The chain rule tells us the derivative of a composite function is dy/dx*dx/dt. dy/dx = 4x + 8 = (4/t) + 8. $dx/dt = -1/t^2$, so $dy/dt = (-4/t^3 - 8/t^2)$

J. Evaluate the function (y ◦ x) = y(x(t)) at t = 0.25.

The easiest way to do this is to determine that t = 0.25 means x = 4, then evaluate at x = 4 to get $y = 2*4^2 + 8*4 + 3 = 32 + 32 + 3 = 67$

Problem 2: "Inspector Gadget"

Gad-Inc (now the largest producer of gadgets in Ohio) still has the production function:

$$Q = 200\ (K)^{0.25}\ (L)^{0.75}$$

where Q = gadgets per hour
L is the number of workers per hour
K is the quantity of capital employed per hour (measured in machines)

A. If Gad-Inc expands from 100 workers and 10 machines to 150 workers and 15 machines what happens to output?

a) Increases by 50% b) Increases by 100% c) Increases by 150%

d) Decreases by 50% e) Cannot be determined.

Easy way: A Cobb Douglas production function whose powers sum to 1 is constant returns to scale, so increasing both inputs by 50% will increase output by 50%.

Hard way: Work it out:
$Q(100,10) = 200 * 10^{1/4} * 100^{3/4} \approx 11246.83$
$Q(150, 15) = 200 * 15^{1/4} * 150^{3/4} \approx 16870.23$

$16870.23/11246.83 \approx 150$, so the increase is one of 50%.

B. What are the formulas for the Marginal Physical Product of Labor and the Marginal Physical Product of Capital at Gad-Inc?

These are the first derivatives:

Marginal Physical Product of Labor = $dQ/dL = 0.75*200*(K/L)^{1/4} = 150(K/L)^{0.25}$
Marginal Physical Product of Capital = $dQ/dK = 0.25*200*(L/K)^{3/4} = 50(L/K)^{0.75}$

C. In a competitive economy the WAGE is the MARGINAL REVENUE PRODUCT of LABOR, MRP(L), which is itself defined as:

MRP(L) = Marginal Physical Product of Labor * Price of Output

If Gad-Inc has 81 workers, 16 machines, and Gadgets are selling for \$0.33 (33 cents) each, wholesale (that is – the price that Gad-Inc is paid), what is the competitive wage rate?

Hint – you will need your answer from part B.

$MRP(L) = 150(K/L)^{0.25} * 0.33 \approx 50(K/L)^{0.25} = \text{wage}$

$\text{wage} = 50(K/L)^{0.25} = 50(16/81)^{0.25} = 50(2/3) = 100/3 = \$33.33$

D. For any multivariable function $y = f(x_1, x_2, ..., x_n)$ elasticity of y with respect to x_i, or the x_i Elasticity of y, is defined as:

$$\epsilon_{y, x} = (\partial y/\partial x_i)(x_i/y) = (\partial y/\partial x_i)(x_i/f(x_1, x_2, ..., x_n))$$

Using this formula, find the elasticity of Output, Q, with respect to Labor, L, as a function of K and L only, using Gad-Inc's production function. (That is, if Q shows up in your formula, substitute.)

Hint – the Cobb-Douglas Production Function is a Constant Elasticity Function, so don't be surprised if your result is VERY SIMPLE.

$\epsilon_{Q,L} = MP_L * (L/Q) = 150\ (K)^{0.25}\ (L)^{-0.25}\ (L)\ (1 / 200)\ (K^{-0.25})\ (L^{-0.75}) = 3/4$

$\epsilon_{QL} = MP_K * (K/Q) = 50(L)^{0.75}\ (K)^{-0.75}\ (K)\ (1 / 200)\ (K^{-0.25})\ (L^{-0.75}) = 1/4$

E. Cheltenham Royal Widgets, Ltd. (with manufacturing facilities in Cheltenham, UK) has the cost function:

Total Costs = $Q^2 + Q + 100$
Where Q = the number of widgets

Find the Average Total Cost and the Marginal Cost of Widgets.

$AC = TC/Q = Q + 1 + 100Q^{-1}$

$MC = d(TC)/dQ = 2Q + 1$

F. For Cheltenham Royal Widgets, find the MINIMUM Average Total Cost.

Critical Point: where $d(AC)/dQ = 0$
$d(AC)/dQ = 1 - 100/Q^2 = 0$ $Q^2 = 100$ $Q = 10$ or -10.

$Q = -10$ doesn't belong in our range, since we are dealing with production, not consumption. So we just need to check we are at a min at $Q = 10$.

$d^2AC/dQ^2 = 200/Q^3 > 0$ for all $Q > 0$, so we have a min.

Note: we could also find the Q that minimizes costs by finding Q where MC = AC.

G. You are considering buying stock in Cheltenham Widget. The stock is expected to pay a dividend of $10.00 every year forever. If the current (and expected future) CONTINUOUSLY compounded interest rate is 5% what is the fair market price of stock in Cheltenham Widget?

Formula for the price of a continuous stream of future profits is:

Value = Annual Payment / Annual interest rate = 10.00 / 0.05 = $200.00

H. You have a budget of $100.00 to spend on Widgets and Gadgets. Gadgets cost $20.00 each and Widgets cost $10.00 each. In (Gadget, Widget) space, which of the following is the equation for your BUDGET line. (Gadgets, G, are on the y-axis and Widgets, W, on the x-axis.)

☞ **a) G = – (½)W + 5**

20G + 10W = 100
2G + W = 10
G = –(1/2)W + 5
Answer is A.

b) G = – 2W + 5

c) G = ½W + 5

d) G = ½W – 5

e) G = 2W – 5