Anwers for Review Quiz #1.
Material Covered. Klein 1, 2; Schaums 1, 2

1. Solve the following system of equations for x, y and z:

$$x + y = 2$$
$$2x + 2y + z = 5$$
$$7x + y + z = 9$$

Answers: x = 1, y = 1, z = 1.

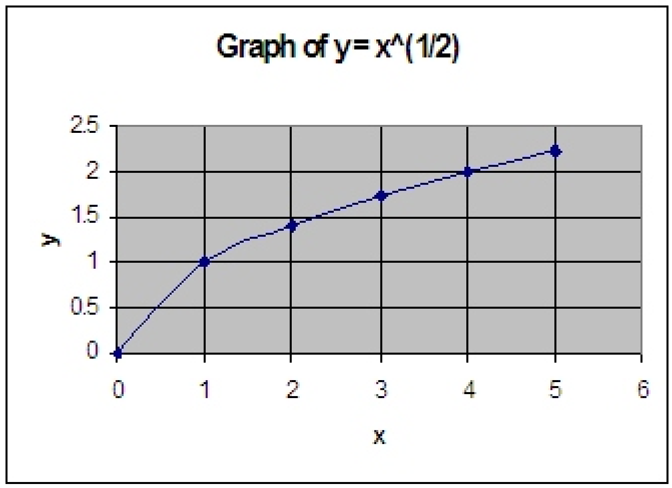
2. Which of the following is the ECONOMISTS inverse of the function $y = 9/x^2$ (i.e. – find x as a function of y, x = f(y))

a) $x = 9/y^2$ b) $x = 3/y^2$ c) $x = 9/y$ d) $x = 3/y$ e) $x = 3/(y^{1/2})$

Answer: e)

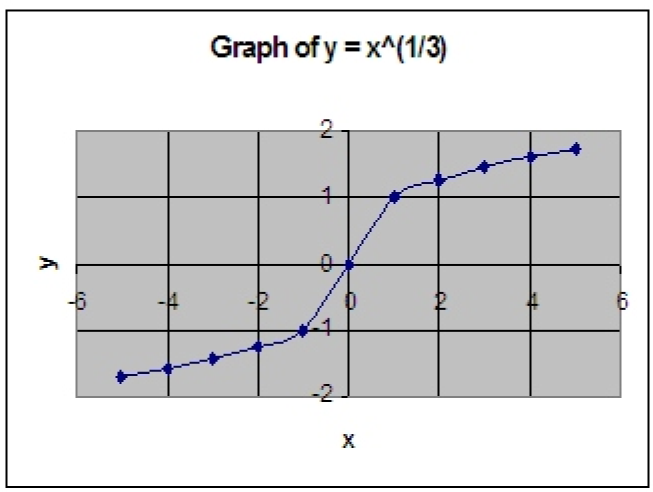
3. Consider the function $y = (x)^{½}$.

A. What is the domain in the real numbers for which this function is defined.
Answer: Since we can't take the square root of a negative number, $x \geq 0$.
B. Sketch the graph of the function over its domain



4. Consider the function $y = f(x) = x^{1/3}$.

A. What is the domain in the real numbers for which this function is defined.
Answer: Since we CAN take the cube root of a negative number, x is any real number.
B. Sketch the graph of the function over its domain.



5. Consider the following supply and demand functions in the Market for Widgets:
Demand: $Q_d = 6 – P^2$ Supply: $Q_s = P$
Find the equilibrium Quantity and Price.
Answer: Q* = P* = 2

Answers for Review Quiz 2:
Material covered in Schaum's 7, 8 (Exponents and Logs, interest compounding).

1. Solve for x: $5e^{x+2} = 120$

Answer: Use the ln to get the x variable down out of the power, then apply algebra rules for lns.

$\ln (5e^{x+2}) = \ln (120) = \ln 5 + (x + 2) \ln e = \ln 120$

$x + 2 = \ln (120/5) = \ln 24$

$x = \ln 24 - 2 \approx 3.18 - 2 = 1.18$

2. Which of the following is equivalent to $\ln(e^x) = y$

a) $x = y$ b) $x = 1/y$ c) $y = 1/x$ d) $e^y = x$ e) $e^y = 1/x$

Answer: $\ln (e^x) = x * \ln e = x$, so the above function is really a) $x = y$.

3. Which will be bigger in 5 years time?

A) $100 invested at 10% annual interest, compounded continuously.

$y(5) = 100*e^{0.1*5} = 100 * e^{0.5} \approx \$164.87$

B) $100 invested at 11% annual interest, compounded annually.

$y(5) = 100*(1.11)^5 \approx 168.51$

So the 11% computed discretely will be larger in 5 years.

4. $\text{Log}_3 27 =$

a) 1 b) 2 c) 3 d) 4 e) 5

Answer: You are being asked to find the answer to $3^x = 27$, so $x = 3$, the answer is c)

5. If the Chinese per capita income was 5,000 in 2000 and the annual rate of growth (compounded continuously) in 6.9%, in what year will the Chinese per capita income be 10,000?

The formula for continuous growth is $y(t) = y(0)e^{0.069\,t}$. We have y(t) and y(0) and we have to find t.

$10{,}000 = 5{,}000\ e^{0.969\,t}$	$2 = e^{0.069\,t}$
Take ln of both sides;	$\ln 2 \approx 0.69 = 0.069t \ln 3 = 0.069\ t$
Divide through by t:	$t = 10.$

Comment – The Chinese growth rate has been running about 7% averaged over the last quarter century, so we actually do expect Chinese incomes to hit about $10,000 per person in the next decade. Politically, this is the per capita income when, in many nations, the population starts to seriously demand political rights. So the next ten years are likely to be (politically) interesting in China.

Review Quiz 3: Schaums 3, Klein

1. Find the first, second, third and fourth derivatives of $y = x^2 + 3x + 1$.

2. Evaluate the following limits

a) $\lim_{x \to 4} \frac{(3x^2 - 5x)}{(x + 6)}$

This one is easy, since the function exists at x = 4, and has the value 2.8. The limit = 2.8

b) $\lim_{x \to -6} \frac{(3x^2 - 5x)}{(x + 6)}$

This one is trickier, since the function if not defined at x = – 6. Basically, we evaluate the numerator AT – 6, then if it is > 0, the limit is + ∞. If the numerator is negative, limit will be – ∞

Math for b) $3 * (-6)^2 - 5(-6) = 108 + 30 = 138$. So the limit is definitely + ∞.

Extra Credit: is the function defined at x = 4? At x = – 6? The function is defined at x = 4, but not at x = – 6.

3. Consider the function $y = (1/x^2)$
Find the x value(s), if any, for which the function is NOT continuous.
Answer: The function is not continuous (doesn't exist) when x = 0.

4. Given the function $f(x) = (2x^7 - x)(3x - 7)$, use the product rule to find the first derivative.
Answer: $df/dx = (14x^6 - 1)(3x - 7) + (2x^7 - x)(3)$

5. Find the first derivative of the function $g(x) = [3(x^7 - x)^7] / (x^7 - x)^{½}$

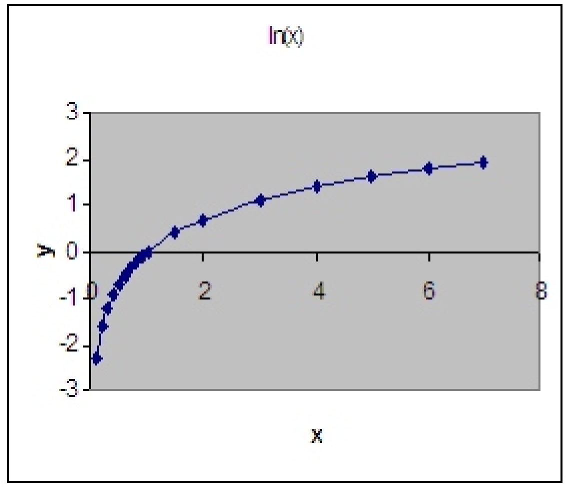
Answer: We can make the job a lot easier if we recognize that $(x^7 - x)$ appears twice in the function. Define $h(x) = (x^7 - x)$, put it into our function, simplify, and use the chain rule.

$g(x) = 3*(h(x))^7 / (h(x))^{½} = 3*(h(x))^{7 - ½} = 3*(h(x))^{13/2}$

$dg/dx = dg/dh * dh/dx = (39/2)(h(x))^{11/2} * dh/dx = (39/2)(x^7 - x)^{11/2} * (7x^6 - 1)$

Answers for Review Quiz 4: Schaum's 4, 9.

1. Concavity and Convexity, and differentiation of the ln and exponential functions:

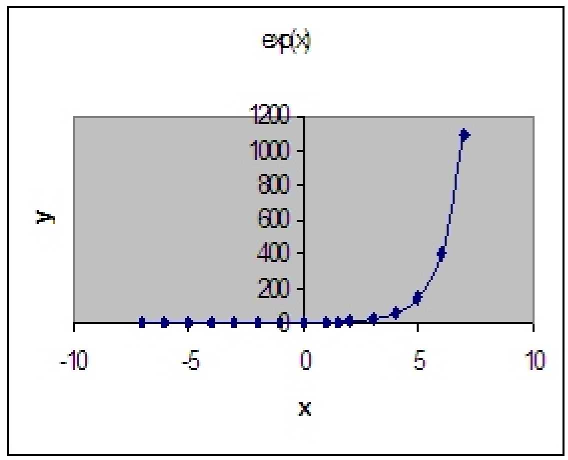


a) Is the function $f(x) = \ln(x)$ concave or convex over the interval $(0, \infty)$? Explain how you know (all credit given for quality of explanation.)

Answer: See the sketch. The slope of the function is going from steep to flat, so the second derivative is negative, and the slope is concave. Let's check the second derivative:

$df/dx = x^{-1}$ $\quad d^2f/dx^2 = -x^{-2}$

When $x > 0$, the second derivative is < 0, so we have a CONCAVE function.



b) Is the function $g(x) = e^x$ concave or convex over the interval $(-\infty, \infty)$? Explain how you know (all credit given for quality of explanation.)

Answer: Again, take a look at the sketch. The slope of the function starts flat but is growing ever steeper, which means we have a convex function. Let's check the second derivative:

$df/dx = e^x$ $\quad d^2f/dx^2 = e^x$

And this is ALWAYS positive, so we must have a CONVEX function.

2. Consider the function $f(x) = 3x^3 - x + (15/9)$

a) Find the critical points

Answer: $df/dx = 9x^2 - 1 = 0$; We solve to get two values: $x = 1/3$ or $x = -1/3$. Now to find $f(1/3)$ and $f(-1/3)$.

$f(1/3) = 3(1/3)^3 - 1/3 + 15/9 = 1/9 - 3/9 + 15/9 = 13/9$ $\qquad (1/3, 13/9)$

$f(-1/3) = 3(-1/3)^3 + 1/3 + 15/9 = -1/9 + 3/9 + 15/9 = 17/9$ $\qquad (-1/3, 17/9)$

b) For each point, determine if it is a local maximum or a local minimum.

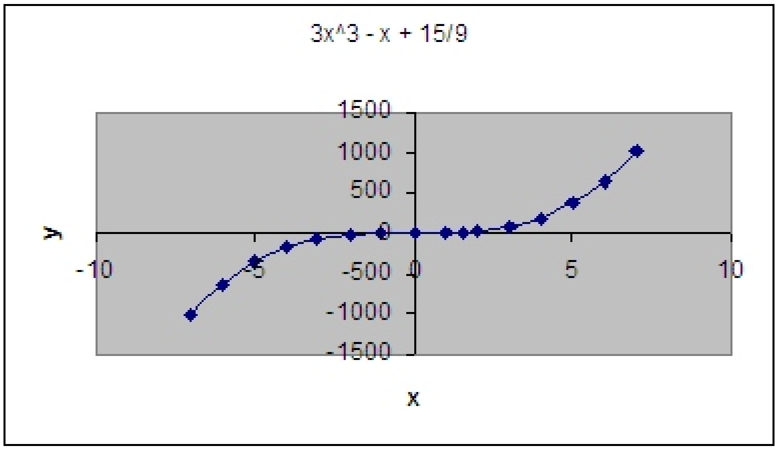
Answer: find the second derivative and evaluate at each point.

$d^2f/dx^2 = 18x$. When $x > 0$, this is > 0, and we have a minimum. $(1/3, 13/9)$ is a min.

$d^2f/dx^2 = 18x$. When $x < 0$, this is < 0, and we have a maximum. $(-1/3, 17/9)$ is a max.

c) Sketch the function over the interval $(-\infty, \infty)$, showing the critical points and the limits as $x \to \infty$ and $x \to -\infty$.

It is a little hard to see in the sketch, but the function passes through (0, 15/19), so it rises up just a teensy bit to 17/19 at x = – 1/3, and falls a teensy bit to 13/19 at x = 1/3.



3. For the Total Cost Function $TC(q) = q^3 - 5q^2 + 60q$, find the Average Cost and the Marginal Cost functions.

$AC = TC/q = q^2 - 5q + 60$
$MC = d(TC)/dq = 3q^2 - 5$

4. For the function in question 3, find the minimum average cost and find the level of q at which this minimum average cost is achieved.

Two ways to solve – a) take first drv of AC and set = 0, b) remember at min AC, AC = MC. In this case a) is the easier method.
a) $d(AC)/dq = 2q - 5 = 0$. $Q = 2.5$; $AC(2.5) = 2.5^2 - 5*2.5 + 60 = 6.25 - 12.5 + 60 = 53.75$.

5. Consider a firm with output q. Total Revenue and Total Cost are both functions of q: TR = f(q) and TC = g(q). Prove that a profit maximizing firm will choose to produce a level of output q, such that Marginal Revenue equals Marginal Cost.
Let TR = Total Revenue, TC = Total Cost, MR = marginal revenue, MC = marginal cost.
Profit = TR – TC
max Profit means finding a point where d(profit)/dq = 0
d(Profit)/dq = d(TR)/dq – d(TC)/dq = MR – MC = 0, so at max profit, MR = MC.

Review Quiz 5: 3, 7, and 9 (Review Quiz), Klein

1. For the Total Cost Function $TC(q) = e^{(0.5\ q)}$ find the Average Cost and the Marginal Cost functions.

$AC = e^{(0.5\ q)} / q$
$MC = d(TC)/dq = 0.5\ e^{(0.5\ q)}$ Use the chain rule

2.For the function in question 1, find the minimum average cost and find the level of q at which this minimum average cost is achieved.

Min AC when AC = MC.
$e^{(0.5\ q)} / q = 0.5\ e^{(0.5\ q)}$ $1/q = ½$ $q = 2$ $AC(2) = e/2$

3. Use logarithmic differentiation to find the first derivative of the following function:
$f(x) = (x^3 – 2)(x^2 – 3)^3$

$\ln f(x) = \ln (x^3 – 2) + 3 \ln (x^2 – 3)$

$d(\ln(f(x)))/dx = f'(x)/f(x) = (3x^2) / (x^3 – 2) + (6x) / (x^2 – 3)$
$= [(3x^2) * (x^2 – 3) + 6x * (x^3 – 2)]/ [(x^3 – 2)(x^2 – 3)^3] = [(3x^2) * (x^2 – 3) + 6x * (x^3 – 2)]/f(x)$
Therefore $df/dx = [(3x^2) * (x^2 – 3) + 6x * (x^3 – 2)]$

4. What is the domain and what is the range of the function $f(x) = \ln x + e^x$
ln x is not defined for $x \le 0$, so the domain is $x > 0$. Range goes from $-\infty$ (at small values of x, ln(x) dominates) to $+\infty$ (at large values of x, e^x dominates.)

5. Prove (show) that the function $y = x^2$ is convex at every value of x.
Convex if second derivative is positive (the first derivative is getting larger and larger).
$dy/dx = 2x$, $d^2y/dx^2 = 2 > 0$, for all x, therefore convex everywhere.

Review Quiz 6: Schaum's 4, 8 and 9 (a review of economic applications)

1. Acme Lobbyists, LLP. is a firm of lobbyists. They use a single input, Labor (L), to produce their output of Billable Hours, (H). The production function for the firm is therefore very simple:

H = L (Basically, each employee is expected to bill every hour to a client.)

Acme pays the market wage, W.

What is Acme's Total Cost as a function of output, H, and W.

Ans: TC(H) = HW

If Billable Hours grows by 10% and the wage paid to Lobbyists grows by 7%, what happens to Acme's Total Costs.

Ans: New TC = 1.1H * 1.07W = 1.1*1.07 HW = 1.17 HW. The TC grow by 17%

2. The art collection of a recently deceased painter has an estimated value of:

$V = 200{,}000 * (1.25)^{t^{\wedge}(2/3)}$.

If the current market interest rate is 6% how long should the executor of the estate wait before selling the collection?

Ans. 15.3 years

Easiest way to solve – find the function that gives the annual rate of return on the investment as a function of t, then solve for t when rate of return = 0.06. What makes this easy is the fact that the current rate of return at time t is equal to the first derivative of the natural log.

Rate of Return = V'(t)/V(t) = d(lnV(t))/dt

Therefore we are solving for t such that d(lnV(t))/dt = 0.06	
$\ln V(t) = \ln 200{,}000 + t^{2/3} \ln(1.25) = 12.21 + .223\, t^{2/3}$	take natural log
$d(\ln V)/dt = 0.223*2/3*t^{-1/3} = 0.149t^{-1/3} = 0.06$	take first derivative
$t^{1/3} = 0.149/0.06 = 2.48$	Algebra
$t = (2.48)^3 = 15.3$	Algebra

3. Determine whether the following functions are convex or concave over the given interval:

Ans. Convex = second derivative is positive, Concave = second derivative is negative

a) $f(x) = e^x$ $\quad -\infty < x < \infty$

$f'(x) = e^x$ $\quad f''(x) = e^x$, and so on. $e^x > 0$, so this function is always convex.

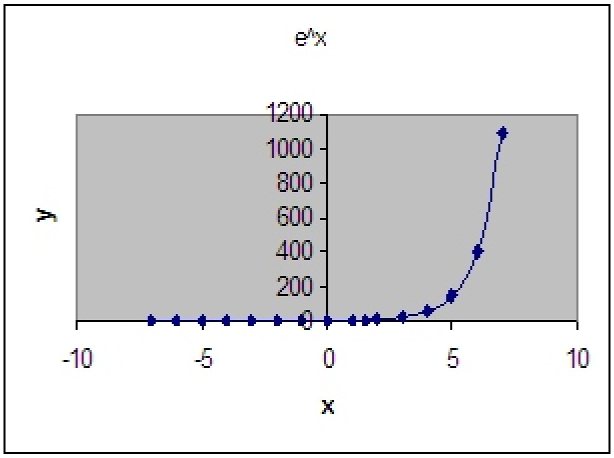
b) $g(x) = \ln x$ $\quad 0 < x < \infty$

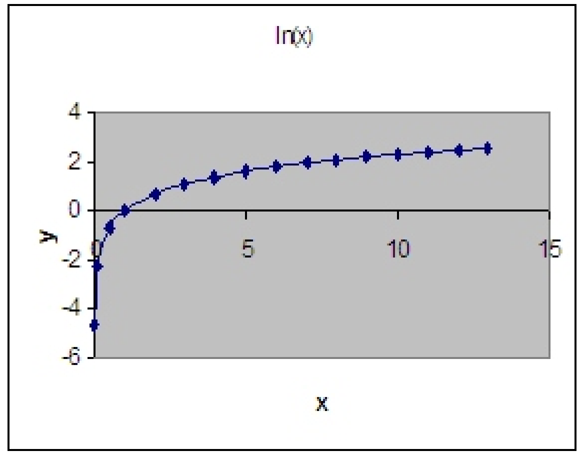
$g'(x) = x^{-1}$ $\quad g''(x) = -x^{-2}$ When x>9, g''(x) < 0, so concave

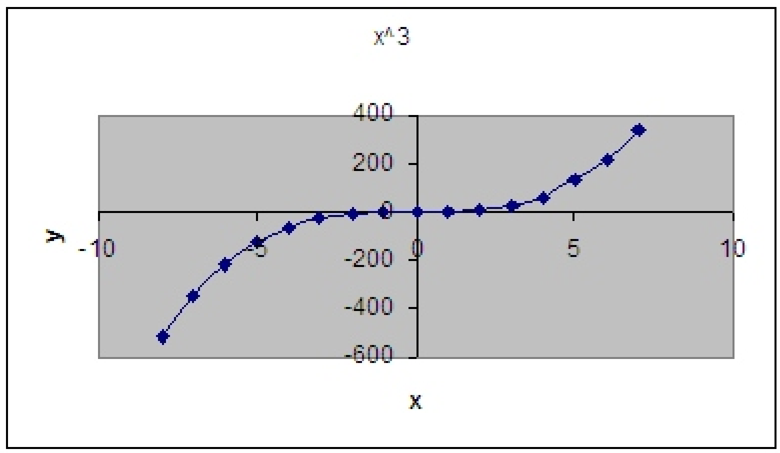
c) $h(x) = x^3$ $\quad -\infty < x < 0$

$h'(x) = 3x^2$ $\quad h''(x) = 6x$ When x < 0, h''(x) < 0, so concave

Sketches:







c) Note how the function is concave for $x < 0$, but convex for $x > 0$.

4. If $y = 3x^2$ and $z = \ln y$, what is dz/dx?
Ans: Chain rule. $dz/dx = dz/dy*dy/dx$. $dz/dy = 1/y = 1/(3x^2)$ $dy/dx = 6x$
$dz/dx = dz/dy*dy/dx = 6x/3x^2 = 2/x$

5. If Demand for Widgets in Stansylvania is given by: $Q_d = 10 – P$

Prove that the Price Elasticity of Demand = $(dQ_d / dP)(P/Q) = d(\ln Q_d) / d(\ln P)$

Approach in 3 steps – 1) find $(dQ_d / dP)(P/Q)$; 2) find $d(\ln Q_d) / d(\ln P)$; 3) show they are equal.
1) $(dQ_d / dP) = – 1$; $(dQ_d / dP)(P/Q) = (– P/Q) = (– P/(10 – P))$
2) $d(\ln Q_d) / d(\ln P) = d(\ln Q_d)/dP * dP/d(\ln P)$ Chain rule
$d(\ln P)/dP = 1/P$ Rule for drv of ln
$dP/d(\ln P) = 1/(d(\ln P)/dP = P$ Inverse function rule
$\ln Q = \ln (10 – P)$ $d(\ln Q)/dP = – 1/(10 – P)$
So $d(\ln Q)/d(\ln P) = [– 1/(10 – P)]*P = (– P/(10 – P))$
3) So both sides are equal to $(– P/(10 – P))$ and our proof is done.

Review Quiz 7: Schaums' Chapters 5 and 6: (Calculus of Multivariable Functions)

1. Given the function: $z = 5x^3 - 3y^2x^2 - \ln(7y^5)$
Find the two first partial derivatives.

$\partial z/\partial x = 15x^2 - 6y^2x$ $\partial z/\partial y = -6yx^2 - (35y^4 / 7y^5) = -6yx^2 - 5/y$

2. For the function in question 1, find the four second partial derivatives

$\partial^2 z/\partial x^2 = 30x - 6y^2$ $\partial^2 z/\partial x\partial y = -12yx$

$\partial^2/\partial y\partial x = -12xy$ $\partial^2 z/\partial y^2 = -6x^2 + 5/y^2$

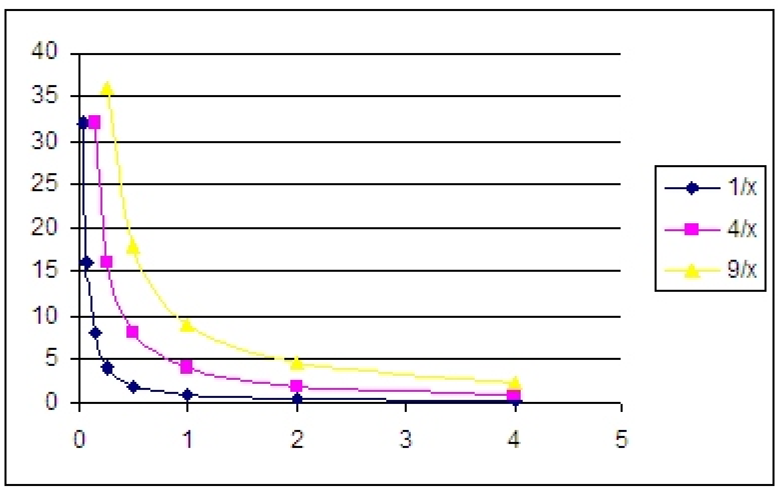
3. Given the Production Function $Q = 36KL - 2K^2 - 3L^2$
Find the Marginal Product of Labor and the Marginal Product of Capital.

Marginal Product of Labor = $\partial Q/\partial L = 36K - 6L$

Marginal Product of Capital = $\partial Q/\partial K = 36L - 4K$

4. Consider the function $f(x, y) = xy$

For this function, sketch a flat x-y graph of the iso-value curves:
$f(x, y) = 1$
$f(x, y) = 4$
and $f(x, y) = 9$.
Step 1: write out the 3 implicit functions and solve for $y = g(x)$:
$xy = 1$ $y = 1/x$
$xy = 4$ $y = 4/x$
$xy = 9$ $y = 9/x$
Step 2: graph these three functions



5. What is the limit of this function as x and y both go to infinity: $\lim_{x, y \to \infty} f(x, y)$?

$\lim_{x, y \to \infty} (xy) = \infty$

In other words, the product of x and y, xy, goes to ∞ if either x or y goes to ∞, so of course it goes to ∞ if they both go to ∞.

Answers for Review Quiz 8: Schaum's 5 and 6 (Especially 6, focus on applications)

1. Given the Production Function $Q = 10\ K^{0.25}\ L^{0.75}$, Find the Isoquants Q = 10 and Q = 20. (Definition of Isoquant – set Q = a fixed value and solve for K as a function of L.)

$10 = 10\ K^{0.25}\ L^{0.75}$ solve for K: $K^{0.25} = L^{-0.75}$
$20 = 10\ K^{0.25}\ L^{0.75}$ solve for K: $2K^{0.25} = L^{-0.75}$

2. For the Production Function in 1, find the Labor Elasticity of Output and the Capital Elasticity of Output.
Remember the formula for elasticity: given f(x,y); Elasticity of f(x,y) wrt x = ∂f/∂x * x/f(x)

Labor Elasticity of Output $= \partial Q/\partial L * L/Q = 2.5(L/K)^{0.75} * (L/10\ K^{0.25}\ L^{0.75})$

Capital Elasticity of Output $= \partial Q/\partial K * K/Q = 7.5(K/L)^{0.25} * (K/10\ K^{0.25}\ L^{0.75})$

3. The definition of constant returns to scale is: $10\ (mK)^{0.25}(mL)^{0.75} = mQ$
Show that the production function in question 1 is constant returns to scale.
Plan: work out the left hand side of the equation and the right hand side until they look the same.
l.h.s: $10\ (mK)^{0.25}(mL)^{0.75} = 10\ (m)^{0.25}\ (K)^{0.25}(m)^{0.75}\ (L)^{0.75} = m(10\ K^{0.25}\ L^{0.75})$
r.h.s. $mQ = m(10\ K^{0.25}\ L^{0.75})$ Q.e.d.

Consider the standard equation for constant percent growth:

$V(P,r,t) = Pe^{rt}$ Where V = Value at t years in the future
P = Principal invested today
r = percent rate of growth

4. Find the three first partials:

$\partial V/\partial P = e^{rt}$

$\partial V/\partial r = Pt\ e^{rt}$

$\partial V/\partial t = Pr\ e^{rt}$

5. Why can't you find ∂V/∂e?

e is NOT a variable, just a number. (e ≈2.71). Can't take the derivative wrt a number b/c numbers never change.

Answers for Review Quiz 9: Schaum's 5 and 6 Optimization

1. Consider the function $z = 3x^2 – xy + 2y^2 – 4x + 12$

Find the critical points for this function and identify them as minima or maxima or saddles or cannot determine.

First order conditions for the Critical Points:

$\partial z/\partial x = 6x – y – 4 = 0$

Using below: $6*4y – y = 4$; $23y = 4$ $\qquad$ $y = 23/4$

$\partial Z/\partial y = – x + 4y = 0$

$x = 4y$ $\qquad$ $x = 23$

$z = 3x^2 – xy + 2y^2 – 4x + 12$

$z = 3(23)^2 – (23)(23/4) + 2(23/4)^2 – 4*23 + 12 = 1440.875$ $\qquad$ $z = 1440.875$

Second order conditions to determine min, max or saddle:

$z_{xx} = 6$ $\qquad$ $z_{xy} = – 1$

$z_{yx} = – 1$ $\qquad$ $z_{yy} = 4$

So the pure partials are both positive, suggesting I might have a minimum.

$z_{xx}*z_{yy} – (z_{xy})^2 = 24 – 1 = 23 > 0$. $\qquad$ Yes, I have a minimum.

2. Consider the function $z = x^3 – 3xy^2 + y$

Find the critical points for this function and identify them as minima or maxima or saddles or cannot determine.

Answer: probably more than one critical point, b/c of the cube function.

$\partial z/\partial x = 3x^2 – 3y^2 = 0$ $\qquad$ $x = \pm y$

$\partial z/\partial y = – 6xy + 1 = 0$ $\qquad$ $xy = 1/6$ $\qquad$ so we know that x and y have the same sign.

$x = (1/6)^{½}$ $\quad$ $y = (1/6)^{½}$ $\qquad$ or $x = – (1/6)^{½}$ $\quad$ $y = – (1/6)^{½}$

For the two positive values: $z = (1/6)^{3/2} – 3 (1/6)^{3/2} + (1/6)^{½} = (1/6)^{½} – 2 (1/6)^{3/2} \approx 0.41 – 0.14 = 0.27$.

For the two negative values $z = – (1/6)^{3/2} + 3 (1/6)^{3/2} – (1/6)^{½} = – 0.27$

The two critical points are: $((1/6)^{½}, (1/6)^{½}, 0.27)$ and $(– (1/6)^{½}, – (1/6)^{½}, – 0.27)$

Second Order Conditions: $z_{xx} = 6x$ $\qquad$ $z_{xy} = – 6y$

$z_{yx} = – 6y$ $\qquad$ $z_{yy} = – 6x$

Both points must be saddles, since z_{xx} and z_{yy} have opposite signs for any value of x

3. In Masonia the production and sale of Milk and Cookies is a government licensed monopoly. The Revenue function is:

$$\text{Total Revenue} = 10Q_C - 1.2Q_MQ_C + 0.1(Q_C)^2 + (Q_M)^2$$

What are the Quantities of cookies (Q_C) and milk (Q_M) which maximize Total Revenue? (Warning – the quantities are not integers).

This is just a find a critical point problem hidden away in an economics question.

First order conditions:

$$\partial(TR)/dQ_C = 10 - 1.2Q_M + 0.2Q_C = 0$$

$$\partial(TR)/\partial Q_M = -1.2Q_C + 2Q_M = 0 \qquad 1.2\,Q_C = 2Q_M \qquad Q_M = 0.6Q_C$$

Substitute: $10 - .72Q_C + 0.2Q_C = 0$

$$.52Q_C = 10 \qquad Q_C \approx 19.23 \qquad Q_M = 0.6Q_C \approx 11.54$$

4. Consider the function $z = exy - e^x - ey$

Find the critical point(s) for this function and identify them as minima or maxima or saddles or cannot determine. (Remember e is a number, approximately equal to 2.71)

NOT a TYPO – treat e as a number.

$\partial z/\partial x = ey - e^x = 0 \qquad ey = e^x \qquad \ln e + \ln y = x \ln e \qquad 1 + \ln y = x$

$\partial z/\partial y = ex - e = 0 \qquad e(x - 1) = 0$ when $x = 1$

Therefore: $1 + \ln y = 1 \qquad \ln y = 0 \qquad$ when $y = 1$

The critical point z value is $z = e*1*1 - e^1 - e*1 = e - e - e = -e$

So our critical point is $(1, 1, -e)$

Max, min or saddle?

$\partial^2 z/\partial x^2 = -e^x \qquad \partial^2 z/\partial x\partial y = e$

$\partial^2 z/\partial y\partial x = e \qquad \partial^2 z/\partial y^2 = 0$

We have a zero value for $\partial^2 z/\partial y^2$, so we cannot determine if we have a min, max or saddle.

Review Quiz 10: Schaum's Chapters 5 and 6 – constrained optimization

1. Constrained optimization – a firm manufactures two goods, x and y. The total cost function is:

$$TC = x^2 + y^2 - xy + 100$$

The firm has a contract to produce 10 units of output:

$x + y = 10$ typo – original says 20.

Using a Lagrangian, find the quantities of x* and y* which will minimize production costs. What is the value of the "shadow price" λ* at x* and y*.

Min $\mathfrak{L} = TC + \lambda(10 - Q) = x^2 + y^2 - xy + 100 + \lambda(10 - x - y)$

f.o.c.:

$\partial\mathfrak{L} / \partial x = 2x - y - \lambda = 0$

$\partial\mathfrak{L} / \partial y = 2y - x - \lambda = 0$

$\partial\mathfrak{L} / \partial \lambda = 10 - x - y = 0$

Solving the algebra: $2x - y = 2y - x$ $3x = 3y$ $x = y$

$10 - 2x = 0$ $x = 5, y = 5$

$10 - 5 - \lambda = 0$ $\lambda = 5$

x* = 5

y* = 5

λ* = 5

2. Practice SETTING UP the problem from an economic situation:
Two inputs are used to produce coffee, capital (coffee makers, cups, etc.) and labor.
The production function for coffee is: $q = 12\, K^{1/4} L^{3/4}$

Where K = quantity of capital required

L = quantity of Labor required, and

q = number of cups of coffee produced per hour

The current WAGE (price of labor) is w = \$1.00 per hour.

The current RENTAL rate for one unit of capital is v = \$0.50 per hour.

The coffee shop owner wants maximize total output while keeping her TOTAL COSTS to \$5.00 per hour or less.

A. Find the "objective function" and the "constraint" and set up the Lagrangian:

Objective Function: $q = 12\, K^{1/4} L^{3/4}$

Constraint: $TC - wL - vK = 05 - L - 0.5K = 0$

Lagrangian: max $\mathfrak{L} = 12\, K^{1/4} L^{3/4} + \lambda(5 - L - 0.5K)$

B. Find the quantities of K and L and that maximize output, subject to the constraint.

First order conditions:

$\partial\mathcal{L}/\partial L = 9\,(K/L)^{1/4} - \lambda = 0$ $\quad\partial\mathcal{L}/\partial K = 3(L/K)^{3/4} - 0.5\lambda = 0$ $\quad\partial\mathcal{L}/\partial\lambda = 5 - L - 0.5K = 0$

Solving: $2/3 = K/L$ $\quad K = 2L/3$

$5 - L - L/3 = 0$ $\quad 5 = 4/3L$ $\quad L = 15/4$ $\quad K = 5/2$

Or if you solved for $q = 2$, $L = 15/2$, $K = 5$.

C. Extra credit: Show that the optimum RATIO of K to L is not a function of the output level.

Well – we showed above that $K/L = 2/3$, independent of quantity (no q in the equation), so the optimum ratio is not a function of the output level.