Review Quiz #1.
Material Covered. Klein 1, 2; Schaums 1, 2

1. Solve the following system of equations for x, y and z:

$$x + y = 2$$
$$2x + 2y + z = 5$$
$$7x + y + z = 9$$

2. Which of the following is the ECONOMISTS inverse of the function $y = 9/x^2$
(i.e. – find x as a function of y, $x = f(y)$)

a) $x = 9/y^2$ b) $x = 3/y^2$ c) $x = 9/y$ d) $x = 3/y$ e) $x = 3/(y^{1/2})$

3. Consider the function $y = (x)^{½}$.

A. What is the domain in the real numbers for which this function is defined.

B. Sketch the graph of the function over its domain

4. Consider the function $y = f(x) = x^{1/3}$.

A. What is the domain in the real numbers for which this function is defined.

B. Sketch the graph of the function over its domain.

5. Consider the following supply and demand functions in the Market for Widgets:

Demand: $Q_d = 6 – P^2$

Supply: $Q_s = P$

Find the equilibrium Quantity and Price.

Review Quiz 2: Schaum's 7, 8 (Exponents and Logs, interest compounding), Klein

1. Solve for x: $5e^{x+2} = 120$

2. Which of the following is equivalent to $\ln(e^x) = y$

a) $x = y$ b) $x = 1/y$ c) $y = 1/x$ d) $e^y = x$ e) $e^y = 1/x$

3. Which will be bigger in 5 years time?

A) \$100 invested at 10% annual interest, compounded continuously.
B) \$100 invested at 11% annual interest, compounded annually.

4. $\text{Log}_3 27 =$

a) 1 b) 2 c) 3 d) 4 e) 5

5. If the Chinese per capita income was 5,000 in 2000 and the annual rate of growth (compounded continuosly) in 6.9%, in what year will the Chinese per capita income be 10,000?

Review Quiz 3: Schaums 3, Klein

1. Find the first, second, third and fourth derivatives of $y = x^2 + 3x + 1$.

2. Evaluate the following limits

a) $$\lim_{x \to 4} \frac{(3x^2 - 5x)}{(x + 6)}$$

b) $$\lim_{x \to -6} \frac{(3x^2 - 5x)}{(x + 6)}$$

Extra Credit: is the function defined at $x = 4$? At $x = -6$?

3. Consider the function $y = (1/x^2)$

Find the x value(s), if any, for which the function is NOT continuous.

4. Given the function $f(x) = (2x^7 - x)(3x - 7)$, use the product rule to find the first derivative.

5. Find the first derivative of the function $g(x) = [3(x^7 - x)^7] / (x^7 - x)^{1/2}$

Review Quiz 4: Schaum's 4, 9, Klein

1. Concavity and Convexity, and differentiation of the ln and exponential functions:

a) Is the function $f(x) = \ln(x)$ concave or convex over the interval $(0, \infty)$? Explain how you know (all credit given for quality of explanation.)

b) Is the function $g(x) = e^x$ concave or convex over the interval $(-\infty, \infty)$? Explain how you know (all credit given for quality of explanation.)

2. Consider the function $f(x) = 3x^3 - x + (15/9)$

a) Find the critical points

b) For each point, determine if it is a local maximum or a local minimum.

c) Sketch the function over the interval $(-\infty, \infty)$, showing the critical points and the limits as $x \to \infty$ and $x \to -\infty$.

3. For the Total Cost Function $TC(q) = q^3 - 5q^2 + 60q$, find the Average Cost and the Marginal Cost functions.

4. For the function in question 3, find the minimum average cost and find the level of q at which this minimum average cost is achieved.

5. Consider a firm with output q. Total Revenue and Total Cost are both functions of q: $TR = f(q)$ and $TC = g(q)$. Prove that a profit maximizing firm will choose to produce a level of output q, such that Marginal Revenue equals Marginal Cost.

Review Quiz 5: 3, 7, and 9 (Review Quiz), Klein

1. For the Total Cost Function $TC(q) = e^{(0.5\ q)}$ find the Average Cost and the Marginal Cost functions.

2.For the function in question 1, find the minimum average cost and find the level of q at which this minimum average cost is achieved.

3. Use logarithmic differentiation to find the first derivative of the following function:
$f(x) = (x^3 - 2)(x^2 - 3)^3$

4. What is the domain and what is the range of the function $f(x) = \ln x + e^x$

5. Prove (show) that the function $y = x^2$ is convex at every value of x.

Review Quiz 6: Schaum's 4, 8 and 9 (a review of economic applications)

1. Acme Lobbyists, LLP. is a firm of lobbyists. They use a single input, Labor (L), to produce their output of Billable Hours, (H). The production function for the firm is therefore very simple:

H = L (Basically, each employee is expected to bill every hour to a client.)

Acme pays the market wage, W.

What is Acme's Total Cost as a function of output, H, and W.

TC(H) =

If Billable Hours grows by 10% and the wage paid to Lobbyists grows by 7%, what happens to Acme's Total Costs.

2. The art collection of a recently deceased painter has an estimated value of:

$V = 200{,}000 * (1.25)^{t\^(2/3)}$.

If the current market interest rate is 6% how long should the executor of the estate wait before selling the collection? (Ans. 15.3 years)

3. Determine whether the following functions are convex or concave over the given interval:

a) $f(x) = e^x$ $\quad -\infty < x < \infty$

b) $g(x) = \ln x$ $\quad 0 < x < \infty$

c) $h(x) = x^3$ $\quad -\infty < x < 0$

(A sketch can help if the second order conditions don't tell you.)(

4. If $y = 3x^2$ and $z = \ln y$, what is dz/dx?

5. If Demand for Widgets in Stansylvania is given by: $Q_d = 10 - P$

Prove that the Price Elasticity of Demand = $(dQ_d / dP)(P/Q) = d(\ln Q_d) / d(\ln P)$

Review Quiz 7: Schaums' Chapters 5 and 6: (Calculus of Multivariable Functions)

1. Given the function:

$$z = 5x^3 - 3y^2x^2 - \ln(7y^5)$$

Find the two first partial derivatives.

2. For the function in question 1, find the four second partial derivatives

3. Given the Production Function $Q = 36KL - 2K^2 - 3L^2$

Find the Marginal Product of Labor and the Marginal Product of Capital.

4. Consider the function $f(x, y) = xy$

For this function, sketch a flat x-y graph of the iso-value curves:

$f(x, y) = 1$
$f(x, y) = 4$
and $f(x, y) = 9$.

5. What is the limit of this function as x and y both go to infinity: $\lim_{x, y \to \infty} f(x, y)$?

Review Quiz 8: Schaum's 5 and 6 (Especially 6, focus on applications)

1. Given the Production Function $Q = 10\,K^{0.25}\,L^{0.75}$, Find the Isoquants Q = 10 and Q = 20. (Definition of Isoquant – set Q = a fixed value and solve for K as a function of L.)

2. For the Production Function in 4, find the Labor Elasticity of Output and the Capital Elasticity of Output.

3. The definition of constant returns to scale is: $10\,(mK)^{0.25}(mL)^{0.75} = mQ$

Show that the production function in question 1 is constant returns to scale.

Consider the standard equation for constant percent growth:

$V(P,r,t) = Pe^{rt}$ Where V = Value at t years in the future
P = Principal invested today
r = percent rate of growth

4. Find the three first partials:

$\partial V/\partial P =$

$\partial V/\partial r =$

$\partial V/\partial t =$

5. Why can't you find $\partial V/\partial e$?

Review Quiz 9: Schaum's 5 and 6 Optimization

1. Consider the function $z = 3x^2 - xy + 2y^2 - 4x + 12$

Find the critical points for this function and identify them as minima or maxima or saddles or cannot determine.

2. Consider the function $z = x^3 - 3xy^2 + y$

Find the critical points for this function and identify them as minima or maxima or saddles or cannot determine.

3. In Masonia the production and sale of Milk and Cookies is a government licensed monopoly. The Revenue function is:

$$\text{Total Revenue} = 10Q_C - 1.2Q_M Q_C + 0.1(Q_C)^2 + (Q_M)^2$$

What are the Quantities of cookies (Q_C) and milk (Q_M) which maximize Total Revenue? (Warning – the quantities are not integers).

4. Consider the function $z = exy - e^x - ey$

Find the critical point(s) for this function and identify them as minima or maxima or saddles or cannot determine. (Remember e is a number, approximately equal to 2.71)

Review Quiz 10: Schaum's Chapters 5 and 6 – constrained optimization

1. Constrained optimization – a firm manufactures two goods, x and y. The total cost function is:

$$TC = x^2 + y^2 - xy + 100$$

The firm has a contract to produce 10 units of output:

$$x + y = 20$$

Using a Lagrangian, find the quantities of x* and y* which will minimize production costs. What is the value of the "shadow price" λ* at x* and y*.

$\mathcal{L} =$

f.o.c.:

$\partial\mathcal{L} / \partial x =$
$\partial\mathcal{L} / \partial y =$
$\partial\mathcal{L} / \partial\lambda =$

$x^* =$
$y^* =$
$\lambda^* =$

2. Practice SETTING UP the problem from an economic situation:
Two inputs are used to produce coffee, capital (coffee makers, cups, etc.) and labor.
The production function for coffee is: $q = 12\, K^{1/4}\, L^{3/4}$

Where K = quantity of capital required
L = quantity of Labor required, and
q = number of cups of coffee produced per hour
The current WAGE (price of labor) is w = \$1.00 per hour.
The current RENTAL rate for one unit of capital is v = \$0.50 per hour.

The coffee shop owner wants maximize total output while keeping her TOTAL COSTS to \$5.00 per hour or less.

A. Find the "objective function" and the "constraint" and set up the Lagrangian:
Objective Function:
Constraint:
Lagrangian: $\mathcal{L} =$

B. Find the quantities of K and L and that maximize output, subject to the constraint.
C. Extra credit: Show that the optimum RATIO of K to L is not a function of the output level.