

Econ 812: Problem Set #1

Highly Recommended: Sit down with Schaum's and work through as many of the problems as you can stand. Especially – you want the logs and exponents to be second nature. I highly recommend doing all of the problems in Chapters 7 and 8.

Graded Problems:

1. Interpreting Supply and Demand Diagrams:

At the turn of the 20th century the United States emerged as a major producer and exporter of iron and steel. The quantities of iron and steel produced in the U.S. and sold overseas increased dramatically. (While production and sale in the domestic market also increased.) In 1890 the U.S. exported 50,000 tons of iron and steel, valued at \$25 million dollars (\$25,000,000) worth of iron and steel. By 1913 the U.S. exported 3 million (3,000,000) tons of iron and steel, valued at \$300 million dollars (\$300,000,000).

Two theories have been proposed by economic historians to explain the rapid growth in U.S. steel exports.

The first theory is that the foreign DEMAND for iron and steel rose between 1890 and 1913. Especially important, according to this theory, was economic growth in Mexico and Canada, which increased the demand for new rail road tracks in these countries. (Both Canada and Mexico did build many miles of new rail road between 1890 and 1913.)

The second theory is that the cost of inputs fell dramatically between 1890 and 1913. Especially important, according to this theory, was the opening in 1892 of new iron ore mines in the Mesabi Range in Minnesota. (Iron ore is the raw material from which iron and steel are manufactured.) The iron ore in the Mesabi is close to the surface and can be mined through relatively inexpensive strip mining, rather than expensive deep shaft mines. As a result of the new, cheaper source for iron ore, the cost per ton of ore dropped from \$83.00 per ton of ore to \$37.00 per ton of ore between 1890 and 1913.

With the information on prices and quantities given, can you determine which effect, increased foreign demand or decreased U.S. input costs, must have been the more important cause of the increase in the quantity of iron and steel exported by the U.S. Explain your reasoning clearly. You may want to use a supply-demand diagram in your explanation.

2. Given the simultaneous equations for two related markets, wheat and rye, find the equilibrium conditions for each market.

Q_{dx} = demand for output x; Q_{sx} = supply of output x; P_x = price of output x.

The Wheat Market:

$$Q_{dw} = 41 - 1.5 P_w + 0.5 P_r$$

$$Q_{sw} = -2.5 + 1.5 P_w$$

The Rye Market:

$$Q_{dr} = 46 + P_w - 2 P_r$$

$$Q_{sr} = -3 + 16 P_r$$

3. Exponents: Solve the following

a) $\log_2 64 = 6$ b) $\log_3 y = 6$ c) $\ln y = .297$
d) $81 = 3x$ e) $1 = e^x$ f) $27 \approx e^x$

4. Take your above equations and convert the logarithms into exponents and the exponents into logarithms.

5. Solve the following for x (some answers will be numerical, some will be functions):

a) $\ln(x + 2)^3 = 3.30$ b) $y = ae^{rx}$

6. Compounding Interest

You buy a house on August 1 of 1999 for 146,000 and you sell the house on August 1 of 2003 for 240,000. What is the annual rate of return on your investment:

a) compounded annually. b) compounded continuously

7. Present Value and depreciation

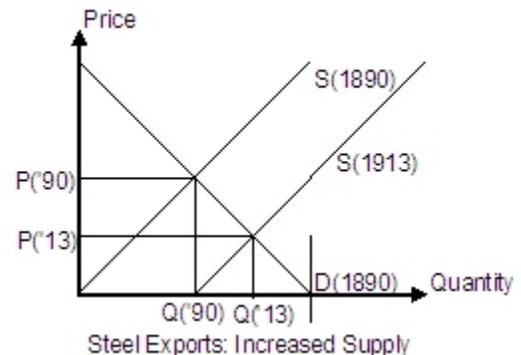
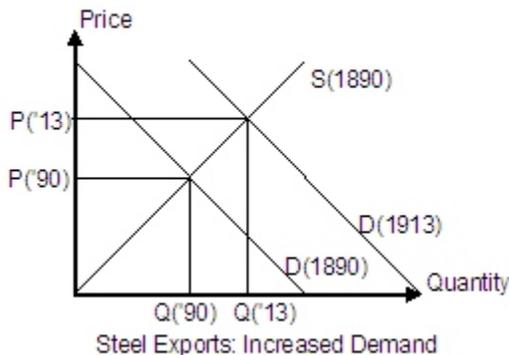
a) You loan your best friend \$100.00, which she will pay back in exactly 2 years. If the current interest rate is 3.5% how much are you giving your friend?

b) You buy a new car. It depreciates by 20% in the first year and 12% for each of the next three years. What will the car be worth in 4 years?

Answer key for Problem Set #1.
 Econ 812
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1. The two theories have the same prediction for quantity – an increase in the demand curve will increase quantity transacted, and so will a drop in input prices and an increase in supply. But the two theories have different implications for the change in prices. When demand increases, prices rise, when supply increases, prices fall. Between 1890 and 1913 prices fell, dramatically. This suggests that the supply increased faster than demand.

Note – the drop in price is proportionately greater than the price drop in the price of ore – suggesting significant technological improvement and decrease in other input prices as well.



2. Solving 4 equations in 4 unknowns: plan

- set $Q_{sx} = Q_{dx}$ for each market. This gives us 2 equations in 2 unknowns.
- Solve for one Price in terms of the other. I solved for $P_w = 18 P_r - 49$
- Substitute back, solve for other price, then substitute into original equations to get quantities.

Solution: $P_w = 15.0935$; $Q_w = 20.14$; $P_r = 3.5607$; $Q_r = 53.97$

3. Exponents

- $(2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2) = 64$
- $y = 3^6 = 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 = 9 \cdot 9 \cdot 9 = 729$
- $\ln y = .297$; $y \approx 1.346$
- TYPO. I meant to write $81 = 3^x$, which has the solution $x = 4$; as written $81 = 3x$; $x = 27$
- $x = 0$
- $\ln 27 \approx x$; $x \approx 3.3$

4. Converting

- $2^6 = 64$
- $3^6 = y$
- $e^{.297} = y$

d) TYPO. $81 = 3^x$ would convert to $\log_3 81 = x$

e) $\ln 1 = x$

f) $\ln 27 = x$

5. a $\ln(x+2)^3 = 3.30;$ $3 \ln(x+2) = 3.30;$ $\ln(x+2) = 1.1;$ $e^{1.1} = x+2$

$$3 = x+2; \quad x = 1$$

b. $y = ae^{rx}; \quad \ln y = \ln a + rx; \quad x = (\ln y - \ln a)/r$

6. A) compounded annually

$$A(t) = A(0)(1+r)^t$$

$$240 = 146 (1+r)^4$$

$$\ln(240/146) = 4 \ln(1+r)$$

$$r \approx 13.23\% \text{ per year}$$

B) compounded continuously

$$A(t) = A(0) e^{rt}$$

$$240 = 146 e^{rt}$$

$$\ln(240/146) = rt$$

$$r \approx 12.426\% \text{ per year.}$$

7. You need to find the difference between 100 today and 100 in 2 years. We might as well compound continuously, it's easier.

$$PV = 100 * e^{(-rt)} = 100 e^{(-0.07)} = 93.24.$$

This means our loan represents a gift of $100 - 93.24 = \$6.7606$.

B) Let P = the original price of the car.

$$\text{Price of car in four years: } P * .8 * .88 * .88 * .88 = .5452 P.$$