Econ 812: Problem Set #2
Spring 2004

1. Given the following functions:
i) find the critical value(s) of x and y.
ii) determine if the function is a minimum or a maximum or inflection point at these value(s).

A) $y = \ln(-x^2 + 8x - 20)$ Chain rule

This is a TRICK QUESTION. The thing above LOOKS like a function, but actually it is not defined for any values of $x \in R^1$.

Proof:
First Derivative and f.o.c.: $dy/dx = (-2x + 8)/(-x^2 + 8x - 20) = 0$
holds when x = 4. But then $(-16 + 32 - 20) = -4$.

Sub back into $y = \ln(-4)$ is NOT DEFINED. Therefore this is not a critical point. What can this mean. Actually what it means is that the function is not defined for ANY value of x, because the function $g(x) = -x^2 + 8x - 20$ is always < 0. (The point $(x,y) = (4, -4)$ is a max of g(x), so g(x) < 0 for all x, and ln (g(x)) is not defined.

B) $y = x^3 * e^{-0.3x}$

Take the ln first, to simplify this problem:

$\ln y = 3 \ln x - 0.3x$ $\quad d(\ln y)/dx = y'/y = 3/x - 0.3$

first order conditions: $y'(x) = 0$ Which is true only if $y'/y = 0$

$y'/y = 3/x - 0.3 = 0 \quad 0.3x = 3 \quad x^* = 10 \quad y^* = 10^3 e^{-3} \approx 49.79$

$y' = (3/x - 0.3)(x^3 * e^{-0.3x}) = e^{-0.3x}(3x^2 - 0.3x^3)$

Second order conditions (Repeat our trick:)

$\ln(y') = -0.3x + \ln(3x^2 - 0.3x^3)$
$d(\ln y')/dx = y''/y' = -0.3 + (6x - 0.9x^2) / (3x^2 - 0.3x^3)$

$= -0.3(3x^2 - 0.3x^3) + (6x - 0.9x^2) / y'$

$y'' = -0.3(3x^2 - 0.3x^3) + (6x - 0.9x^2) = -0.9x^2 - 0.3x^3 + 6x - 0.9x^2$
$y'' = 6x - 1.8x^2 - 0.3x^3 = 60 - 180 - 300 = -420 < 0.$
We have a maximum at (10, 49.79)

C) $y = (3x - 1)(5x + 2)^2$

$dy/dx = 150x^2 + 25x - 14$

quadratic formula for the roots: $x = -(b \pm (b^2 - 4ac)^{½}) / 2a$

$a = 150 \quad b = 25 \quad c = - 14$

$x = -(25 \pm (625 + 4*150*14)^{½}) / (300) = -(25 \pm 95) / 300$

$x = - 120 / 300 = - 0.4 \quad y = 0$

$x = 70 / 300 = 7/30 = 0.233.... \quad y = - 3$

$y'' = 300x + 25 \quad y''(- 0.4) = - 120 + 25 < 0 \quad (- 0.4, 0)$ is a MAX

$y'' = 300x + 25 \quad y''(7/30) = 70 + 25 = 95 > 0 \quad (7/30, 3)$ is a MIN

D) $x^2 - y^3 = 12$ (use implicit differentiation)

$2x\, dx - 3y^2\, dy = 0$

$dy/dx = 2x/3y^2$

Therefore $dy/dx = 0$ when $x = 0$ and $y = -(12)^{1/3} \approx - 2.29$

To find the second derivative, we will use the Total differential of the first derivative:

$d(dy/dx) = (\partial y' / \partial x)dx + (\partial y' / \partial y)dy = (2/3y^2)dx - (4x/3y^3)dy$

$d^2y/dx^2 = (2/3y^2) - (4x/3y^3)dy/dx = (2/3y^2) - (4x/3y^3)(2x/3y^2) = (2/3y^2) - (8x^2 / 9y^5)$

Evaluated at $(0, - 2.29)$. Only the first term survives, since $x = 0$, so $y'' = 2/(3*2.29^2) \approx 0.127 > 0$

We have a minimum.

2. Given the Total Cost function $TC = 10Q^3 - 50Q^2 + 300Q$

find the average cost function, the marginal cost function, the critical point at which average cost is minimized (find both Q and AC at this point.)

$AC = 10Q^2 - 50Q + 300$

$MC = 30Q^2 - 100Q + 300$

At the minimum AC, AC = MC is one way: $10Q^2 - 50Q + 300 = 30Q^2 - 100Q + 300$

Or take first drv of the AC: $20Q - 50 = 0$ when $Q = 2.5$; $AC = 237.5$ at $Q = 2.5$

3. Given the Production function $Q = 81K^{0.25}L^{0.75}$ and the output quantity = 1944. Find the

Marginal Rate of Technical Substitution, dK/dL, and evaluate at K = 3, L = 2.

$\partial Q/\partial K = (81/4)(L/K)^{0.75}$

$\partial Q/\partial L = (81*3/4)(K/L)^{0.25}$

$dK/dL = (\partial Q/\partial L)/(\partial Q/\partial K) = 3K/L$.   This is the envelope theorem.

At K = 3, L = 2, we have MRT = 4.5

4. Find the Hessian for each of the following functions – the Hessian is the matrix of second drvs

$\partial^2 f/\partial x^2$	$\partial^2 f/\partial x\partial y$	$\partial^2 f/\partial x\partial z$
$\partial^2 f/\partial x\partial y$	$\partial^2 f/\partial y^2$	$\partial^2 f/\partial y\partial z$
$\partial^2 f/\partial x\partial z$	$\partial^2 f/\partial y\partial z$	$\partial^2 f/\partial z^2$

a) $f(x,y,z) = xyz$ $\partial f/\partial x = yz$ $\partial f/\partial y = xz$ $\partial f/\partial z = xy$

$\partial^2 f/\partial x^2 = 0$	$\partial^2 f/\partial x\partial y = z$	$\partial^2 f/\partial x\partial z = y$
$\partial^2 f/\partial x\partial y = z$	$\partial^2 f/\partial y^2 = 0$	$\partial^2 f/\partial y\partial z = x$
$\partial^2 f/\partial x\partial z = y$	$\partial^2 f/\partial y\partial z = x$	$\partial^2 f/\partial z^2 = 0$

b) $f(x,y,z) = e^{3x} + \ln(x+2y+3^z) - x^2y^3$

The 3^z makes this a tricky problem. See Schaum's 9.1.2

$\partial f/\partial x = 3e^{3x} + (1)/(x + 2y + 3^z) - 2xy^3$

$\partial f/\partial y = 0 + 2/(x + 2y + 3^z) - 3x^2y^2$

$\partial f/\partial z$ = See Schaum's 9.1.2.

f_{xx} =	f_{xy}	f_{xz}
f_{xy}	f_{yy}	f_{yz}
f_{xz}	f_{yz}	f_{zz}

5. Find the differential for each function

a) $y = \ln x + 3x$ $dy = (x^{-1} + 3)dx$

b) $y = (x+3)(x^2 - 2)$ $y = x^3 + 3x^2 - 2x - 6$ $dy = (3x^2 + 6x - 2)dx$

6. Given the Demand function $Q = 200 – 16P + 0.01Y$
where P = price and Y = income.

Find the general formula for the Price Elasticity of Demand and Income Elasticity of Demand.

Price Elasticity of Demand = $(dQ/dP * P/Q)$

$dQ/dP = – 16$. Therefore
Price Elasticity of Demand = $(– 16)*P/Q = – 16P/(200 – 16P + 0.01Y)$

Income Elasticity of Demand = $0.01Y/(200 – 16P + 0.01Y)$

7. Problem # 7 was moved to Pbm Set 3.