Answers for Econ 812: Problem Set #7 Econ 812: Spring 2004

Varian 13.3, 13.4, and 13.5

1. Varian 13.3
An industry consists of a large number of firms, each of which has a cost function of the form:

$$c(w_1, w_2, y) = (y^2 + 1)w_1 + (y^2 + 2)w_2$$

a) find the average cost curve of the firm and describe how it shifts as the factor price w_1/w_2 changes.

$$AC = c/y = y(w_1 + w_2) + (w_1 + 2 w_2)/y = y(w_1/w_2 + 1) + (w_1/w_2 + 2)/y$$
$$= (w_1/w_2)((y^2 + 1)/y) + ((y^2 + 2)/y)$$

$$dAC/d(w_1/w_2) = ((y^2 + 1)/y)$$

b) Find the short-run supply curve of an individual firm. I interpret this question in 3 possible ways:
Assume we have two inputs, L_1 and L_2, for which the prices are w_1 and w_2.
If we are at long run equilibrium at some output level y*, we can assume from the above equation that $L_1 = (y^*)^2 + 1$ and $L_2 = (y^*)^2 + 2$. Further, we assume in the short run all firms must stay in business. Our options concern what we can alter in the short run, L_1 or L_2 or both.

Let's start with the assumption that both L_1 and L_2 are fully flexible in the SR. In this case we only have to worry about the fixed cost:

$$C = (w_1 + w_2)y^2 + w_1 + 2 w_2$$
Fixed Costs = $w_1 + 2 w_2$ Variable Costs = $(w_1 + w_2)y^2$

So the supply curve = the marginal cost curve:

$$\text{Supply} = P = MC = dc/dy = 2y w_1 + 2y w_2 = 2(w_1 + w_2) y$$

subject to the constraint that P > Avg variable Costs = $(w_1 + w_2)y$ > MC for all y, so no shut down constraint.

Note:
We could also have made the assumption that either L_1 or L_2 is fixed – in which case we set $L_1 = (y^*)^2 + 1$ or $L_2 = (y^*)^2 + 2$. Then when we take the first drv we treat y* as a constant.

C) Find the long run industry supply curve.

In the long run, we will run along the minimum AC(y) = MC(y) line. So set MC = AC and solve for y*. This result tells us the most efficient size for a firm. From this y* we can determine the minimum AC = MC(y*). Since input prices are fixed at w_1 and w_2, the LR supply curve will simply be a horizontal line at this P = MC(y*) = AC(y*).

$MC = 2(w_1 + w_2)\, y = AC = (w_1 + w_2)\, y + (w_1 + 2\, w_2)/y$

Algebra
$(w_1 + w_2)y = (w_1 + 2\, w_2)/y$

$(w_1 + w_2)y^2 = (w_1 + 2\, w_2)$

$y^2 = (w_1 + 2\, w_2) / (w_1 + w_2)$

$y^* = [(w_1 + 2\, w_2)/(w_1 + w_2)]^{1/2}$

Substituting back: $MC = 2(w_1 + w_2) * [(w_1 + 2\, w_2)/(w_1 + w_2)]^{1/2} = 2\, [(w_1 + 2\, w_2)*(w_1 + w_2)]^{1/2}$.

This is the long run supply curve.
Simplifies to: $P = (4w_1^{\,2} + 12w_1w_2 + 8w_2^{\,2})^{1/2}$

d) Describe the input requirements of a single firm:
The form of the cost functions suggests we have two inputs, L_1 and L_2, for which the prices are w_1 and w_2. If we are at long run equilibrium at some output level y*, we can assume from the above equation that $L_1 = (y^*)^2 + 1$ and $L_2 = (y^*)^2 + 2$.

2. Varian 13.4
Farmers produce corn from land and labor. The labor cost, in dollars, to produce y bushels of corn is $c(y) = y^2$. There are 100 identical farms which all behave competitively.

a) What is the individual farmer's supply curve for corn.

$c'(y) = 2y = P$

b) What is the market supply curve for corn?
Need the Inverse Supply to find the market supply (since Y = 100y)

$p = 2y \leftrightarrow y = p/2$

Therefore Y = 100 y = 50p, or p = Y/50.

c) Suppose that the demand curve is D(p) = 200 – 50p. What is the equilibrium p and y?

Equilibrium: Y(p) = D(p)

Y(p) = 50p = 200 – 50p = D(p)

$100p = 200$

$p^* = 2$. $Y^* = 100$, $y^* = 1$.

d) What is the equilibrium RENT on the land?
Remember $c(y) = y^2$
Well, Rent = Profit = Revenue – Costs = $p^*y^* - c(y^*) = 2.00 - 1.00 = \$1.00$

3. Varian 13.5 – the answer to this problem is incomplete.
Consider a model where the U.S. and England engage in trade in umbrellas. The representative firm in England produces the export model umbrella according to the production function f(K,L) where K and L are the amounts of capital and labor used in production. Let *r* and *w* be the price of K and L respectively, in England. Let c(w, r, y) be the cost fcn associated with the prod'n fcn f(K,L). Suppose that initially the equilibrium price of umbrellas is p* and the equilibrium output is y*. Assume for simplicity that all of the export model umbrellas are exported, that there is no production of umbrellas in the U.S., and that all markets are competitive.

Production = f(K,L); Cost = c(w, r, y); given w, r yields a y*, p*. $p^* = c'(w, r, y^*)$

A) England decides to subsidize the prod'n and export of umbrellas by imposing an export subsidy, s, on each umbrella, so that each exporter receives (p + s) per umbrella. What size of import tax, t, should the U.S. impose so as offset the subsidy (keep y at y*).

Answer: set t = s, and then the firms will face exactly the original costs and revenue that they face in the absence of government intervention. The only effect will be to transfer sy* from English taxpayers to U.S. taxpayers.

B) Let p_E = price that the English producers receive
Let $p_{US} = p_E + t = p^*$ (so that Americans continue to purchase y*)

Therefore – the tax must equal the gap in the marginal cost seen by English producers when capital costs r – s and the original marginal cost seen by English producers.

$$t(s) = p^* - p_E = c'(w, r, y^*) - c'(w, r - s, y^*)$$

C) Calculate the expression for t'(s), involving the conditional factor demand function for capital, K(w, r, y)

d) Suppose that the production function exhibits crs. How does this simplify dt/ds?

e) Suppose that capital is an inferior factor of production in umbrella making. What is unusual about the tariff t(s) that will offset the capital subsidy in England?