Problem Set #8. Due April 20
Varian
14.1, 14.5, 14.6.
Extra credit – 14.15

14.1
Inverse demand $p(y) = 10 - y$
Monopolist has $y = 4$.

The MC = 0 up to 4 units. If MC = 0 for more than four units, the quantity that WOULD max profits is 5, and the price would be 5. (Soln to max $(10 - y)*y$. Profit = 25). Since only have 4 to sell, will sell all 4 at the max price, which is 6. Profit = 4*6 = 24. (Essentially, MC for units above 4 = ∞)

Competitive market would actually produce the SAME result – with supply limited to 4, the optimal price would be 6.

If the monopolist had 6 units available, they would burn one unit, sell 5 at $5.00 per unit. The competitive market would sell all 6 units at $4.00 per unit.

14.5
Inverse demand $p(y, t)$; t is a demand shifter. Assume monopolist with MC = C, some constant.

Derive function for $y^M = f(t)$, how monopolist outcome shifts as demand curve shifts.

Start with the objective function:

$$\pi^M = p(y, t)*y - Cy$$

f.o.c.: $$d\pi^M /dy = p(y, t) + \partial p(y, t) /\partial y * y - C = 0$$

Solving for y: $y = (C - p(y, t)) / \partial p/\partial y$

Now we sub in an explicit functional form for $p(y, t) = a(y) + b(t)$
I interpret this formula as saying that a is a fcn of y and b is a fcn of t.

$$p(y, t) = a(y) + b(t) \qquad \text{therefore } \partial p/\partial y = da/dy$$

And we get: $$y = (C - a(y) - b(t)) / da/dy$$

$$y = (C - a(y)) / (da/dy) - b(t) / (da/dy)$$

14.6
Monopolist facing demand curve $D(p) = 10/p$
Monopolist has constant $MC = c$.
Find y^M, the monopolist's profit maximizing outcome:

We know the profit max'ing y will have the characteristic that $MR(y) = MC(y)$

$y = 10/p$ Therefore inverse demand $= p = 10/y$

Total Revenue $= (10/y)y = 10$
$MR = 0$

Therefore MR < MC, and the monopolist should shut down. $y^M = c/20$.

Thanks to Matt for pointing out my typo. Also – notice that this demand function has a constant price elasticity of demand = -1, so MR = 0.

14.15
A single monopolist.
$MC = c$
Demand exhibits constant elasticity, ϵ.
Ad Valorem tax. (Percentage tax) $P_S = P_D (1 - \tau)$

Considering change to unit tax: $P_S = P_D - t$

Given this market structure, from the consumer's perspective, what t is equivalent to τ. (Find $t(\tau)$ st that $P_D(\tau) = P_D(t)$)

Plan – Find the demand function
– solve for monopolist price and quantity in terms of t and in terms of τ.
– Set $P_D(\tau) = P_D(t)$) and solve for t in terms of τ.

– Finding the Demand Function:
We don't have an explicit functional form for the demand function; we only know that the demand is constant elasticity (CES). But this does give us the functional form:

Definition of Elasticity: $\epsilon = d(\ln y) / d(\ln P_D)$

we will get such a result if we have the function $\ln y = \epsilon (\ln P_D)$.

Which is the same relationship as: $y = (P_D)^{\epsilon}$

Inverse demand $P_D = y^{1/\epsilon}$

– Solving for monopolist Price and Quantity under the two tax regimes:

$$\text{Profit} = P_M * y - c*y$$

Under *ad valorem* tax regime:
$P_M = P_D(1 - \tau)$
Profit $= [P_D(1 - \tau)] * y - cy = (y^{1/\epsilon})(1 - \tau)y - cy$

Profit $= (y^{(1 + \epsilon)/\epsilon})(1 - \tau) - cy$

first order conditions: $d\pi/dy = ((\epsilon + 1)/\epsilon)(y^{1/\epsilon})(1 - \tau) - c = 0$

Solving for $y^M(\tau)$: $y^M(\tau) = [(c\epsilon) / (1 - \tau)(1 + \epsilon)]^\epsilon$

And now we turn to consider the unit tax situation:

$$\text{Profit} = P_M * y - c*y$$
$$P_M = P_D - t$$

Profit $= [P_D - t] * y - cy = ((y^{1/\epsilon}) - t)y - cy$

$= ((y^{(\epsilon + 1)/\epsilon}) - ty - cy$

first order conditions:

$$d\pi/dy = ((\epsilon + 1)/\epsilon)\, y^{1/\epsilon} - t - c = 0$$

Solving for $y^M(t)$: $y^M(t) = [(t\epsilon + c\epsilon) / (\epsilon + 1)]^\epsilon$

– Set $P_D(\tau) = P_D(t))$ and solve for t in terms of τ.

$$y^M(\tau) = [(c\epsilon) / (1 - \tau)(1 + \epsilon)]^\epsilon = y^M(t) = [(t\epsilon + c\epsilon) / (\epsilon + 1)]^\epsilon$$

Raise both sides to $1/\epsilon$ to get rid of the power, then cancel out $(\epsilon + 1)$ in both denominators and ϵ in both numerators.

$[(c) / (1 - \tau)] = t + c$ Solves to $t = (c\tau) / (1 - \tau)$.