Econ 812: Problem Set #9 Econ 812: Spring 2004

Varian Chapter 15 (Game Theory)
15.1, 15.6, 15.8

15.1
Calculate all the Nash Equilibria for the game of matching pennies.
The game is found in Varian on page 261. None of the pure strategies are best responses for both players, so we must have mixed strategy equilibria. The rule for understanding when a mixed strategy is a Nash Equilibrium is that it must be mixed in such a proportion that choosing any of the pure strategies elements would yield the same results as the mixed strategy is yielding. (Intuitively, the marginal benefit of each element strategy must be equal.)

Let's start with the obvious mixed strategy – 50% heads, 50% tails for both players.

Well, if the other guy is playing 50/50, then ALL of my choices have a 50% chance of winning – picking pure heads, picking pure tails, or in fact any mixed strategy. So this is an equilibrium response – and each "element" in the mix would have the same % success as pursuing the mixed strategy (assuming the other guy continues to pursue 50/50).

What about another mixed strategy, say 70% heads, 30% tails:
Now the best response to my strategy is to pick all heads (or all tails – depending on whether I'm row or column.) But then 70/30 is no longer MY best response, and we aren't at an equilibrium.

So 50/50 is the only Nash Equilibrium of this game.

15.6

Game of Chicken:		Column (Ted)	
		Stay	Swerve
Row (Bill)	Stay	– 3, – 3	2,0
	Swerve	0,2	1,1

a) The pure strategies:
(Bill stays, Ted stays)
    If Bill is going to Stay, then Ted's Best Response would be Swerve
    If Ted is ging to Stay, then Bill's Best Response would be to Swerve.
    So this is NOT and equilibrium.
(Swerve, Stay)
    If Ted is going to Stay – Bill's Best Response is to Swerve.
    If Bill is going to Swerve – Ted's Best Response is to Stay.
    So this IS a Nash Equilibrium
(Stay, Swerve)
    This is also a Nash Equilibrium, by symmetry.

(Swerve, Swerve)

If Ted is going to Swerve, Bill's Best Response would be to Stay.
If Bill is going to Swerve, Ted's Best Response would be to Stay.
So this is NOT a Nash Equilibrium.

Our two pure strategies are (Swerve, Stay) and (Stay, Swerve). You can see the problem of perfect information is the real problem in this game.

B) Mixed strategy – this is just "matching pennies" again, only both players want very much to not agree. I know by symmetry, therefore, that (½. ½) is the only possible mixed strategy equilibrium. However I am going to work the problem out explicitly – to show how it is done.

If Ted is pursuing a mixed strategy, Bill might have a Best Response mixed strategy. Then Ted's Strategy has to be a Best Response to Bill's and we've got it. We double check by making sure that each element in Bill's Best Response is also a Best Response (not an equil. BR, though, b/c Ted's BR is not the mixed strategy he started with.)

Making it explicit:
Let p_T be the probability that Ted picks STAY. Then $(1 - p_T)$ is the probability that Ted picks Swerve. Our strategy is now: $(?, p_T)$

Now we need a way to figure out Bill's pay-off as a function of his strategy (expressed as p_B) and $p_{T.}$
It helps to START with the pure strategies:
Pay-off if Bill picks Stay: $E(\text{Stay}, p_T) = (p_T)(-3) + (1 - p_T)(2)$
Pay-off if Bill picks Swerve: $E(\text{Swerve}, p_T) = (p_T)(0) + (1 - p_T)(1)$

So we can now express Bill's expected payoff as a function of his mix of choices. (The key to this game, as with many, is being able to write out the objective function explicitly.)

$$E(p_B, p_T) = p_B * E(\text{Stay}, p_T) + (1 - p_B) * E(\text{Swerve}, p_T)$$
$$= p_B * [(p_T)(-3) + (1 - p_T)(2)] + p_B * [(p_T)(0) + (1 - p_T)(1)]$$

NOW we can express the problem of finding the mixed strategy as an OPTIMIZATION problem. Solving the optimization problem will give us the reaction function for Bill (symmetric for Ted.) We can then use these two reaction functions to find the equilibrium.

Bill's Problem:

$$\text{Max}_{pB}\ E(p_B, p_T) = p_B * (-3p_T + 2 - 2p_T) + p_B * (0 + 1 - p_T)$$
$$= 3\, p_B - 6\, p_B\, p_T$$
$$= p_B\, (3 - 6\, p_T)$$

We would now solve by finding the first derivative:

$dE/d\, p_B = (3 - 6\, p_T)$ Notice how p_T is not a function of p_B in the reaction function. That is what makes it Bill's REACTION to a fixed choice by Ted.

But the first order conditions fall apart, because we have a linear function. The first derivative = 0 only if $3 – 6\ p_T = 0$. So instead we think about the function for three situations:

$(3 – 6\ p_T) > 0$	$p_T < ½$	In this case $E(p_B, p_T) = p_B (3 – 6\ p_T)$ is maximized by picking the largest possible p_B, which is $p_B = 1$. Not a mixed strategy. Because of the symmetry, we can also assume that this means that our mixed strategy equilibrium will NOT feature probabilities less than 1/2.
$(3 – 6\ p_T) < 0$	$p_T > ½$	In this case $E(p_B, p_T) = p_B (3 – 6\ p_T)$ is maximized by picking the smalles possible p_B, which is $p_B = 0$. Not a mixed strategy. Because of the symmetry, we can also assume that this means that our mixed strategy equilibrium will NOT feature probabilities greater than 1/2.
$(3 – 6\ p_T) = 0$	$p_T = ½$	In this case $E(p_B, p_T) = p_B (3 – 6\ p_T)$ is always the same, zero, no matter what Bill does. So we know that $p_T = ½$ is our only candidate for a mixed strategy. Now we need to ask – to what strategy set by Bill (p_B = ?) is $p_T = ½$ the best response. By our analysis above, plus symmetry, we know it is $p_B = ½$.

So we conclude the only mixed strategy EQUILIBRIUM is (½, ½).

C) What is the probability that both teenagers survive?
Obviously, given either of the two pure equilibria, both survive.
In the mixed situation – Each outcome has an equal chance of happening, so we know (stay, stay) happens 25% of the time. So both survive 25% of the time.

Most generally – for any strategy set (p_B, p_T) we can add the probabilities to the game:

Game of Chicken:	Column (Ted)	
	Stay (p_T)	Swerve ($1 – p_T$)

Row (Bill)	Stay (p_B)	– 3, – 3 Probability =(p_B)(p_T)	2,0 Probability = (p_B)(1 – p_T)
	Swerve (1 – p_B)	0,2 P = (1 – p_B)(p_T)	1,1 P = (1 – p_B)(1 – p_T)

And we can work out the chance of being at (Stay, Stay) for any mixed strategy, not just the equilibrium strategy.

15.8
The game is:

		Column		
		Left (L)	Middle (MC)	Right (R)
Row	Top (T)	1, 0	1, 2	2, – 1
	Middle (MR)	1, 1	1, 0	0, – 1
	Bottom (B)	– 3, – 3	– 3, – 3	– 3, – 3

B) which of Row's strategies is strictly dominated, no matter what Column does?
If Column picks L – Row's preference ordering of strategies is T = MR >B [1 = 1 > – 3]
If Column picks MC: Row's preference ordering is T = MR > B [1 = 1 > – 3]
If Column picks R: Row's preference ordering is T > MR > B [2 > 0 > – 3]
Bottom is ALWAYS LAST so it is strictly dominated by both other choices

B) Which of Row's strategies is weakly dominated:
Go back to the preference orderings

When Column picks L or MC: Row is indifferent between T and MR.

When Column picks R, Row prefers T to MR.

So MR is weakly dominated by T.

C) Which of Column's strategies is strictly dominated, no matter what Row does:
If Row picks T: Column's preference ordering is: MC > L > R [2 > 0 > – 1]
If Row picks MR: Column's preference ordering is: L > MC > R [1 > 0 > – 1]
If Row picks B: Column's preference ordering is: L = MC = R [– 3 = – 3 = – 3]
None of Column's strategies are strictly dominated, no matter what Row does, but R is weakly dominated.

D) If we eliminate Column's dominated strategies, are any of Row's strategies weakly dominated? (I am confused and suspect a typo – since none of Column's strategies are strictly dominated.)
I read this problem two ways – first, eliminate Column's WEAKLY dominated strategy, which is R. In this case M is still the weakly dominated strategy for Row.

Second, What if we assume the author has confused Row and Column – in this case, we elimate ROW's strictly dominated strategy – B. Column is now picking based only on the pay-offs that show in the first two rows. In this case the R is now strictly dominated. Nothing is weakly dominated.

I apologize for not checking out this problem more carefully before assigning it.